8104: Recitation #N?

Principal agent problem (MWG 14.B)

- Imagine that the owner of a firm (the principal) wishes to hire a manager (the agent) for a project.
- The project's profits are affected by the manager's actions.
- What happens if the actions of the agent are observable? What happens if they are not?
- The manager's contract should be constructed to incentivize the agent to take the correct actions.

Set-up

- Let e denote the manager's $e \in \{e_L, e_H\}$ action choice.
  - Manager's effort must not be perfectly deducible from observation of e.
- Let π denote the project's observable profits.
  - $\pi \in [\pi, \bar{\pi}]$ and has a conditional density function $f(\pi \mid e)$.
  - Any potential realization of π can arise following any given effort choice by the manager.
  - Assume that the distribution of π conditional on e_H first-order stochastically dominates the distribution conditional on e_L. Ie, $F(\pi \mid e_L)$ and $F(\pi \mid e_H)$ satisfy $F(\pi \mid e_H) \leq F(\pi \mid e_L)$ at all $\pi \in [\pi, \bar{\pi}]$, with strict inequality on some open set $\Pi \subset [\pi, \bar{\pi}]$.
- Manager's utility function: $u(w, e) = v(w) - g(e)$ with $v'(w) > 0, v''(w) \leq 0$, and $g(e_H) > g(e_L)$.
- Risk-neutral owner that maximizes his expected return (project's profits less any wage payments made to the manager).

The Optimal Contract when Effort is Observable

- Suppose the owner chooses a contract to offer the manager that the manager can accept or reject.
- A contract is $(e, w(\pi))$.
- Assume that the manager has an outside option of $\bar{u}$.
- The optimal contract for the owner solves:

$$\max_{e \in \{e_H, e_L\}, w(\pi)} \int (\pi - w(\pi)) f(\pi \mid e) d\pi$$
$$\text{s.t.} \quad \int v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \bar{u}.$$

- We could also write the problem in 2 steps:
  1. For each choice of e what is the best $w(\pi)$ to offer the manager?
  2. What is the best choice of e?
- Step 1:
  - Given e, maximizing the total revenue for the owner is the same as:

$$\operatorname*{Min}_{w(\pi)} \int w(\pi) f(\pi \mid e) d\pi$$
$$\text{s.t.} \quad \int v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \bar{u}$$

    Remark: The constraint will always bind.

- The first order condition with respect to $w(\pi)$:

$$\frac{1}{v'(w(\pi)}=\gamma$$

- If the manager is risk averse, the unique solution yields $w(\pi)=\bar{w}$ st:

$$(\bar{w})=v^{-1}(\bar{u}+g(e))$$

Note that since $g(e_H)>g(e_L)$, the wage paid to the manager should be higher if e_H is optimal.

- Interpretation: This finding is a risk-sharing result. The risk-neutral owner should fully insure the risk-averse manager against any risk in his income stream. Since effort is observable there is no problem here.
- Result 2: If the manager is risk neutral $v(w)=w$. Then, $w(\pi)=w$ is also a solution. In particular any $w(\pi)$ such that $\int w(\pi)f(\pi \mid e)d\pi=\bar{u}+g(e)$ solves the problem.

- Step 2:
  - The owner optimally specifies the effort level $e \in \{e_H, e_L\}$ that maximizes his expected profits less wage payments:

$$\int \pi f(\pi \mid e)d\pi - v^{-1}(\bar{u}+g(e))$$

Proposition 14.B.1: In the principal-agent model with observable managerial effort, an optimal contract specifies that the manager chooses the effort e^* that maximizes $\left[\int \pi f(\pi \mid e)d\pi - v^{-1}(\bar{u}+g(e))\right]$ and pays the manager a fixed wage $w^*=v^1\left(\bar{u}+g\left(e^*\right)\right)$. This is the uniquely optimal contract if $v''(w)<0$ at all w.

- The optimal contract accomplishes two goals:
  1. It specifies an efficient effort choice by the manager.
  2. Fully insures him against income risk.

The Optimal Contract when Effort is Not Observable: Risk neutral manager

- *Proposition 14.B.2*: In the principal-agent model with unobservable managerial effort and a risk-neutral manager, an optimal contract generates the same effort choice and expected utilities for the manager and the owner as when the effort is observable.
- Idea of the Proof: The owner offers a compensation schedule of the form $w(\pi)=\pi-a$ Where α is a constant. By doing so, the incentives of the manager align with the incentives of the owner. The manager chooses the level of effort that maximizes the expected profits, and the owner chooses $alpha$ such that $\alpha^*=\int \pi f\left(\pi \mid e^*\right) d\pi - y\left(e^*\right)-\bar{u}$. Hence, with a compensation scheme of $w(\pi)=\pi-\alpha$ both the owner and the manager get exactly the same payoff as when the effort is observable.
- The risk-bearing concern is absent, the owner can still achieve the same outcome as when the effort is observable.

The Optimal Contract when Effort is Not Observable: Risk-averse manager

- In this case, whenever the full observability contract would involve the high-effort level the presence of non-observable actions leads to a welfare loss.
- Incentives for high effort can be provided only at the cost of having the manager face risk.
- The optimal incentive scheme solves:

$$\begin{aligned}
&\underset{w(\pi)}{\operatorname{Min}} \int w(\pi)f(\pi \mid e)d\pi \\
&\text{s.t. (i)} \int v(w(\pi))f(\pi \mid e)d\pi - g(e) \geq \bar{u} \\
&\quad\text{(ii) } e=\underset{\tilde{e}}{\operatorname{argmax}} \int v(w(\pi))f(\pi \mid \tilde{e})d\pi - g(\tilde{e})
\end{aligned}$$

where (i) is the manager participation constraint and (ii) is the manager's incentive compatibility constraint.

- Step 1: For each choice of e what is the best $w(\pi)$ to offer the manager, given that we want the manager to give e effort?
  - Suppose that the owner wishes to implement effort level e_L.
    * In this case, the owner optimally offers the manager the fixed wage payment $w^* = v^1\left(\bar{u} + g\left(e^*\right)\right)$.
    * This is the same payment he would offer if contractually specifying effort e is observable.
    * The manager selects e_L since his wage payment is unaffected by his effort.
  - Suppose that the owner wishes to implement effort level e_H.
    * The incentive compatibility constraint can be written as:

$$\int v(w(\pi)) f\left(\pi \mid e_H\right) d\pi - g\left(e_{II}\right) \geq \int v(w(\pi)) f\left(\pi \mid e_L\right) d\pi - g\left(e_L\right)$$

    * Letting $\gamma \geq 0$ and $\mu \geq 0$ denote the multipliers on constraints (i) and (ii), $w(\pi)$ must satisfy:

$$\frac{1}{v'\left(w(\pi)\right)} = \gamma + \mu\left[1 - \frac{f(\pi \mid e_L)}{f(\pi \mid e_H)}\right]$$

    * Lemma 14.B.1: In any solution both $\gamma > 0$ and $\mu > 0$.
    * Using this result:

$$w(\pi) > \hat{w} \quad \text{if} \quad \frac{f\left(\pi \mid e_L\right)}{f\left(\pi \mid e_H\right)} < 1$$
$$w(\pi) < \hat{w} \quad \text{if} \quad \frac{f\left(\pi \mid e_L\right)}{f\left(\pi \mid e_H\right)} > 1$$

    * The optimal compensation scheme pays more than $\hat{w}$ for outcomes that are statistically relatively more likely to occur under $\boldsymbol{e_H}$ than under e_L in the sense of having a likelihood ratio $\left[f\left(\pi \mid e_L\right) / f\left(\pi \mid e_H\right)\right]$ less than 1.
    * It offers less compensation for outcomes that are relatively more likely when e_L is chosen.
    * By structuring compensation in this way, it provides the manager with an incentive for choosing e_I instead of e_L.
    * Remark: Compensation is not necessarily monotonically increasing in profits! for the optimal compensation scheme to be monotonically increasing, it must be that the likelihood ratio $\left[f\left(\pi \mid e_L\right) / f\left(\pi \mid e_I\right)\right]$ is decreasing in π. As π increases, the likelihood of getting profit level π if the effort is e_H relative to the likelihood of the effort is e_L must increase.
    * This property is not implied by first-order stochastic dominance:

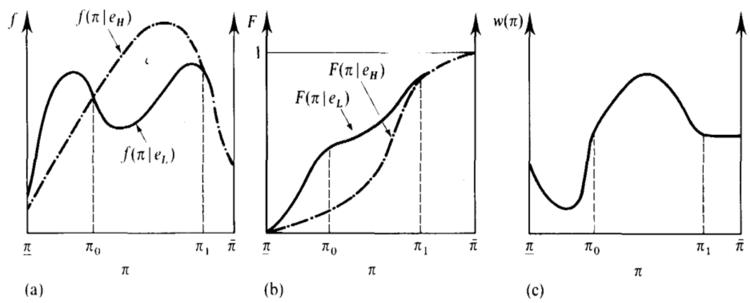

Figure 14.B.1
A violation of the monotone likelihood ratio property.
(a) Densities.
(b) Distribution functions. (c) Optimal wage scheme.

In this example, increases in effort serve to convert low-profit realizations into intermediate ones but have no effect on the likelihood of very high-profit realizations.

    * Note that given the variability that is optimally introduced into the manager's compensation, the expected value of the manager's wage payment must be strictly greater than his (fixed) wage payment in the observable case.
  - Conclusion: the wage payment when implementing e_L is exactly the same as when the effort is observable, whereas the expected wage payment when the owner implements e_H under nonobservability is strictly larger than his payment in the observable case. Thus, in this model, nonobservability raises the cost of implementing e_H and does not change the cost of implementing e_L. Whenever the optimal effort level with observable effort would be e_H, nonobservability causes a welfare loss.

- Step 2: The owner compares the change in expected profits from the two effort levels with the difference in expected wage payments in the contracts that optimally implement each of them to decide.