RECITATION 3 : MICRO B102

<u>Definition:</u> A price/allocation (p, c) is a competitive equilibrium for a given specification of ownership $\{e_1, \dots, e_I\}$ if :

i) $p \in \mathbb{R}^M_+$

ii) $c_i \in C_i \quad \forall i$

iii) $\sum_i c_{i,m} \leq e_m \quad \forall m$ } feasible

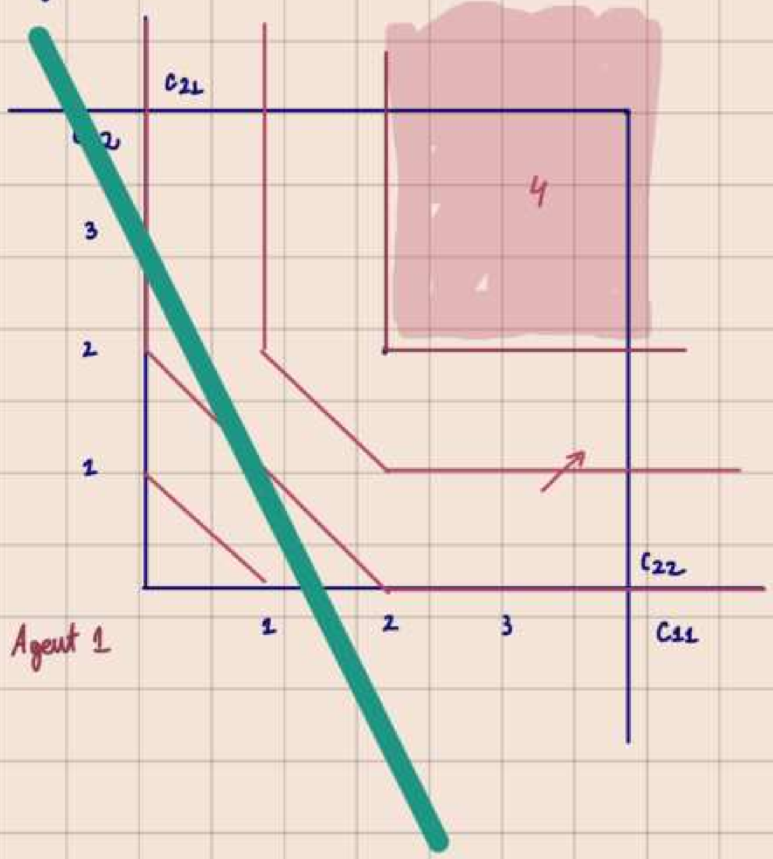
iv) $\forall i, \; \sum_m p_m c_{im} \leq \sum_m p_m e^i_m$ } affordable

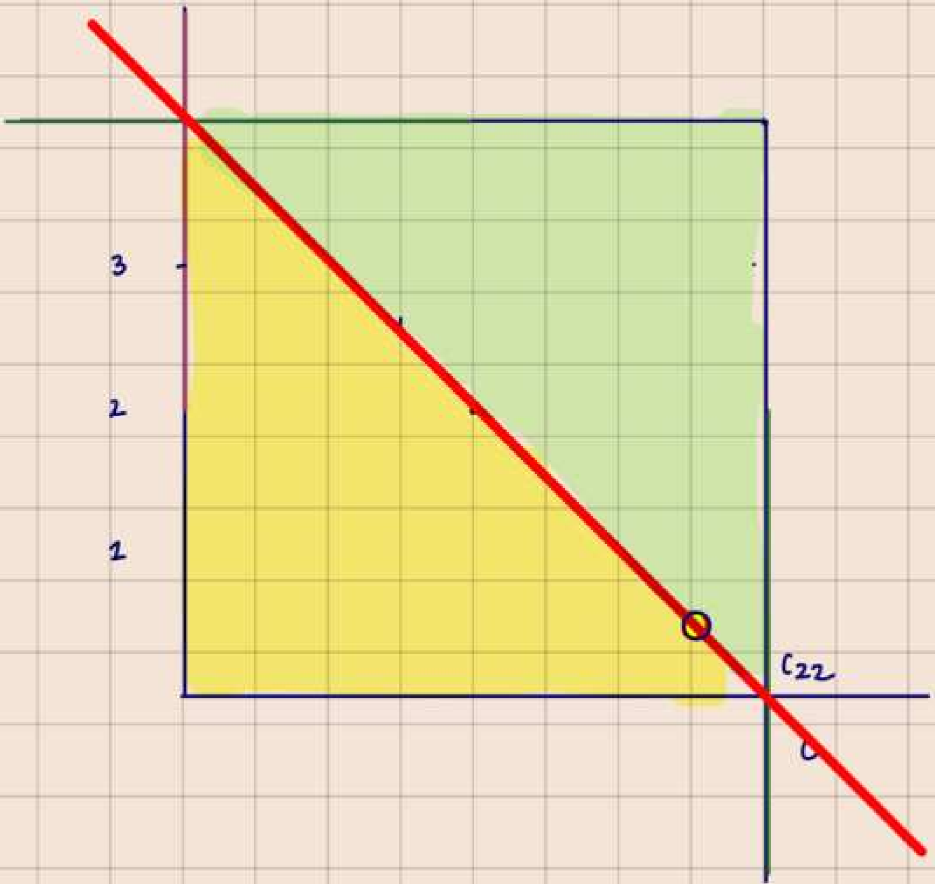
v) $\forall i$ and $\hat{c}_i$ st $\hat{c}_i \succ_i c_i$, $\sum_m p_m \hat{c}_{im} > \sum_m p_m e_{im}$ } Individually optimal

strictly prefered bundles are not affordable.

Drawing for agent 1 :

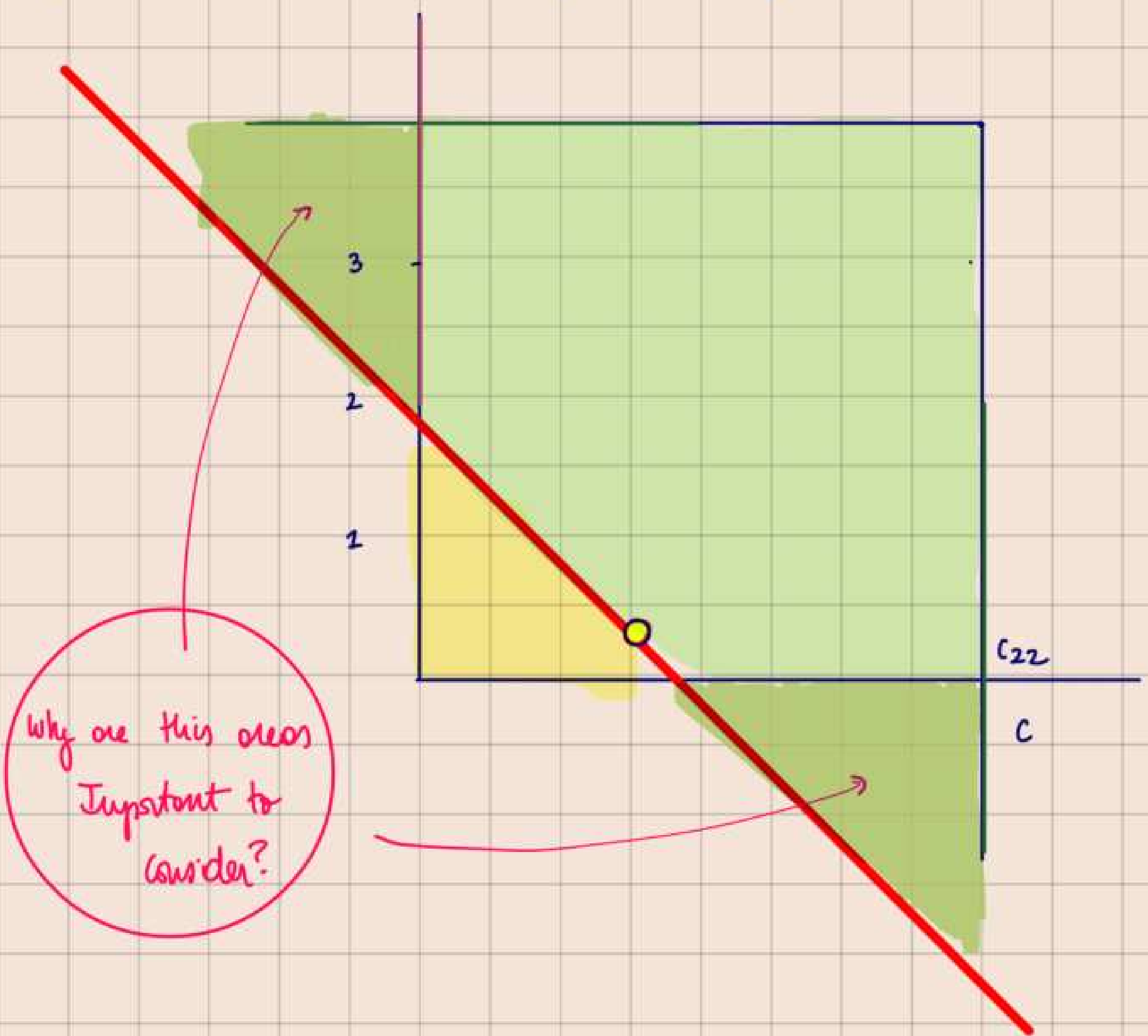
$P_1 > P_2$



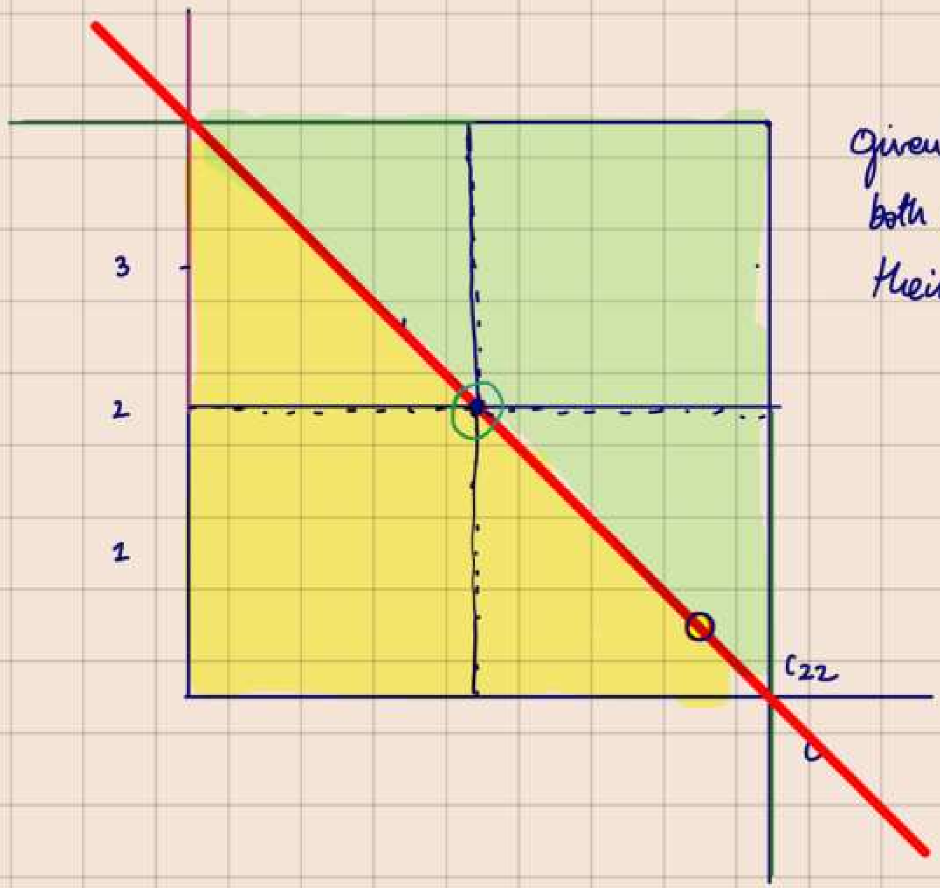


Given an endowment equal to ● All the green area is what the agent 2 can afford and all the yellow area is what agent 1 can afford.

If we move the endowment, but not prices:

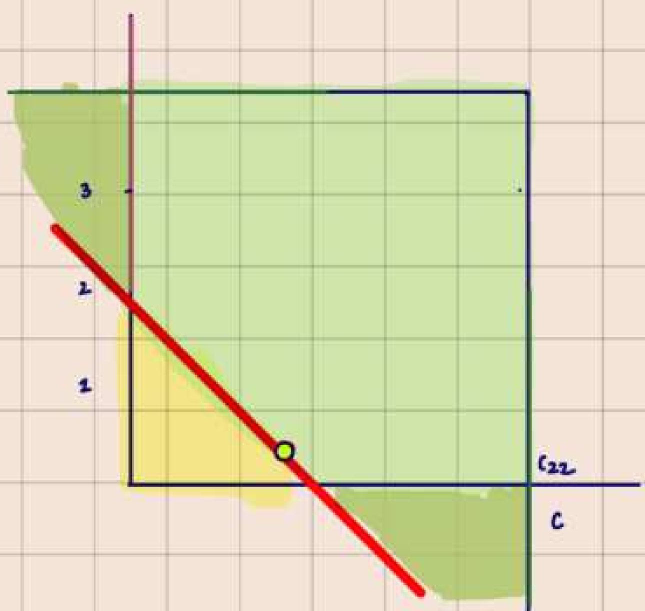


The previous analysis showed us which allocations were affordable. In order to know which allocations are optimal for the agent (given endowment and taking prices as given) we need to include preferences into the edgeworth box.



Given this endowments both agents can reach their satiation point.

If one agent is richer than the other one (suppose agent 2 is richer than agent 1):

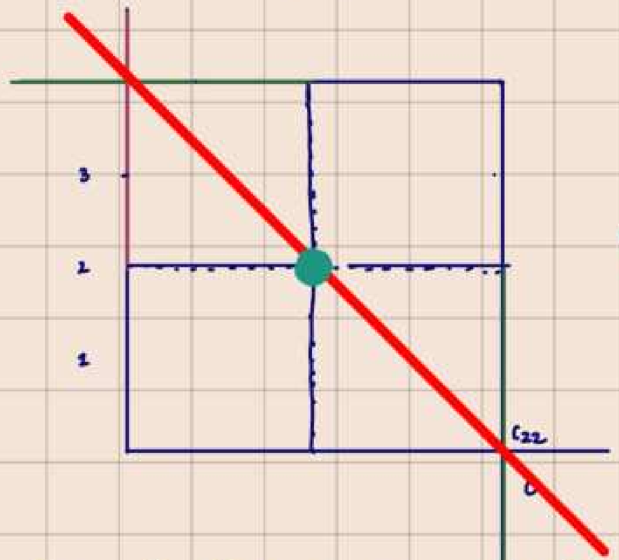


Only agent 2 can reach his satiation point. (But we are going to take advantage of that.)

To gain intuition, analyse some relevant cases for Agent 1.

If prices are equal.

CASE 1: Agent 1 is rich enough, that he can afford to get saciated:

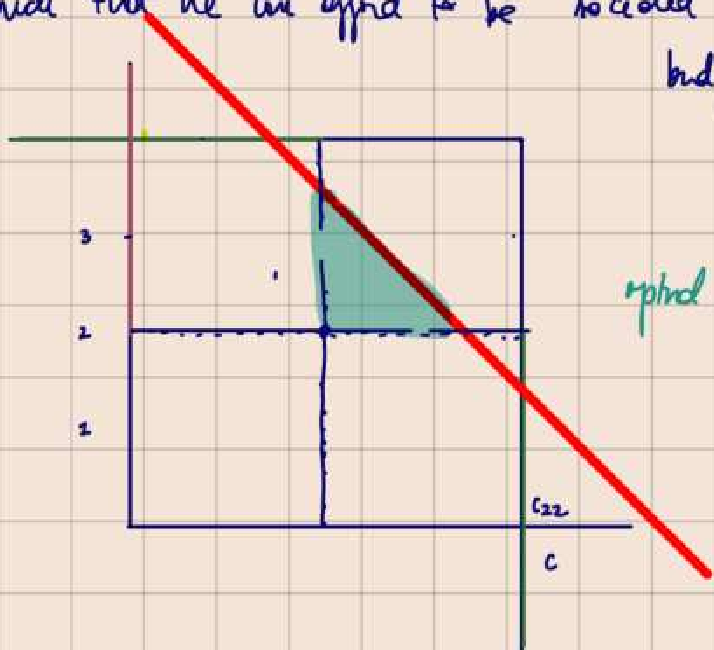


optimal demand.

Agent 1 is poor (cannot afford to get saciated):

optimal demands

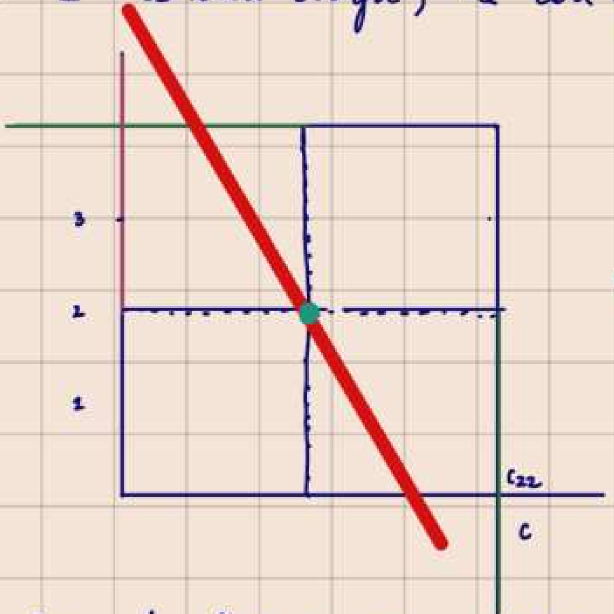
Agent is so rich that he can afford to be saciated without spending all his budget.



optimal demands.

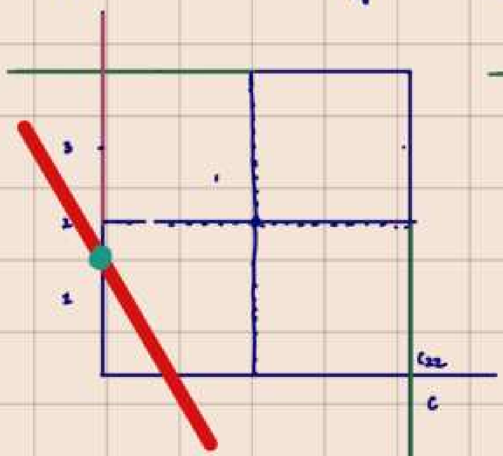
where prices are not equal. Assume $p_1 > p_2$. (good 1 is more expensive)

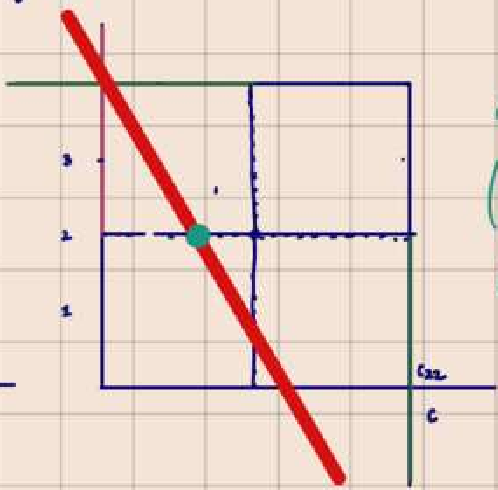
CASE 1: Agent 1 is rich enough, he can afford to get sociated.



optimal demand.

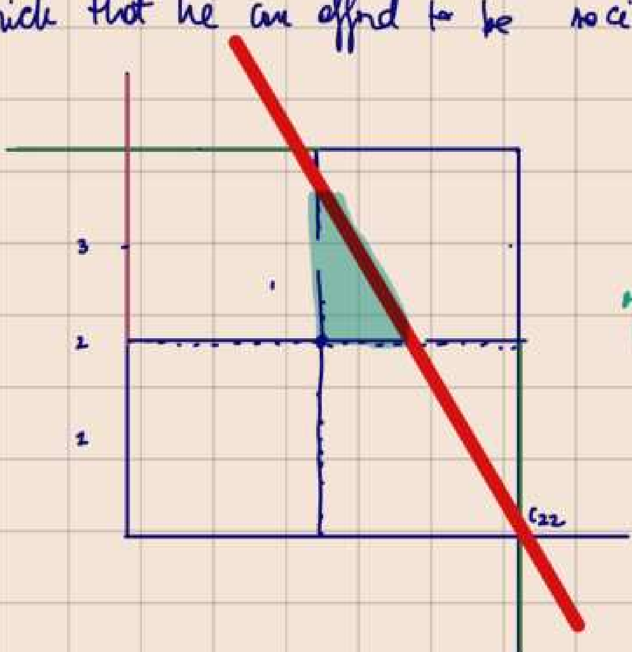
Agent 1 is poor (cannot afford to get sociated





optimal demands (get sociated from the chepeast good then exhaust budget)

Agent 1 is so rich that he can afford to be sociated without spending all his budget.



optimal demands.

Demand for agent i as a function of prices and endowments;

for $p_1, p_2 \in \mathbb{R}_+$.

If $p_1 = p_2$:

does not need to spend everything

$$c_i^*(p_1, p_2, e_{i1}, e_{i2}) = \begin{cases} (x,y) \in \mathbb{R}_+^2 \text{ st: } x,y \geq 2 \text{ and } x+y \leq e_{i1}+e_{i2} & \text{if } e_{i1}+e_{i2} \geq 4 \\ (x,y) \in \mathbb{R}_+^2 \text{ st: } \max\{x,y\} \leq 2 \text{ and } x+y = e_{i1}+e_{i2} & \text{if } e_{i1}+e_{i2} < 4 \end{cases}$$

will spend everything

If $p_1 > p_2$: good 1 is more expensive

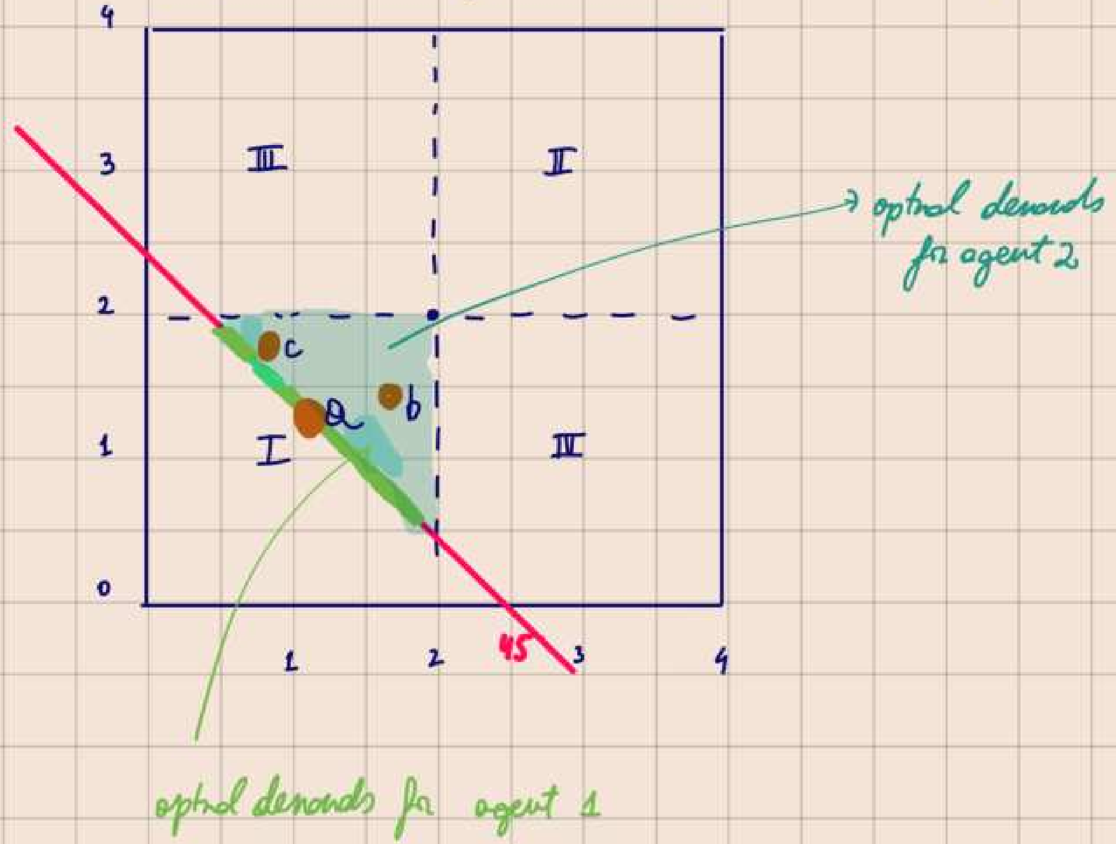
$$c_i^*(p_1, p_2, e_{i1}, e_{i2}) = \begin{cases} \left(0, \frac{p_1 e_{i1} + p_2 e_{i2}}{p_2}\right) & \text{if } \frac{p_1 e_{i1} + p_2 e_{i2}}{p_2} \leq 2 \\ \left(\frac{p_1 e_{i1} + p_2 e_{i2} - 2p_2}{p_2}, 2\right) & \text{if } \frac{p_1 e_{i1} + p_2 e_{i2}}{p_2} > 2 \text{ and } \frac{p_1 e_{i1} + p_2 e_{i2} - 2p_2}{p_1} \leq 2 \\ (x,y) \text{ where } x,y \geq 2 \text{ and } p_1 x + p_2 y \leq p_1 e_{i1} + p_2 e_{i2} & \text{if } \frac{p_1 e_{i1} + p_2 e_{i2} - 2p_2}{p_1} > 2. \end{cases}$$

If $p_2 > p_1$:

$$c_i^*(p_1, p_2, e_{i1}, e_{i2}) = \begin{cases} \left(\frac{p_1 e_{i1} + p_2 e_{i2}}{p_1}, 0\right) & \text{if: } \frac{p_1 e_{i1} + p_2 e_{i2}}{p_1} < 2 \\ \left(2, \frac{p_1 e_{i1} + p_2 e_{i2} - 2p_1}{p_1}\right) & \text{if: } \frac{p_1 e_{i1} + p_2 e_{i2}}{p_1} > 2 \text{ and } \frac{p_1 e_{i1} + p_2 e_{i2} - 2p_1}{p_2} \leq 2 \\ (x,y) \text{ where } x,y \geq 2 \text{ and } p_1 x + p_2 y \leq p_1 e_{i1} + p_2 e_{i2} & \text{if } \frac{p_1 e_{i1} + p_2 e_{i2} - 2p_1}{p_2} > 2. \end{cases}$$

Just saying agents maximize their utility given prices is not enough!

Example:



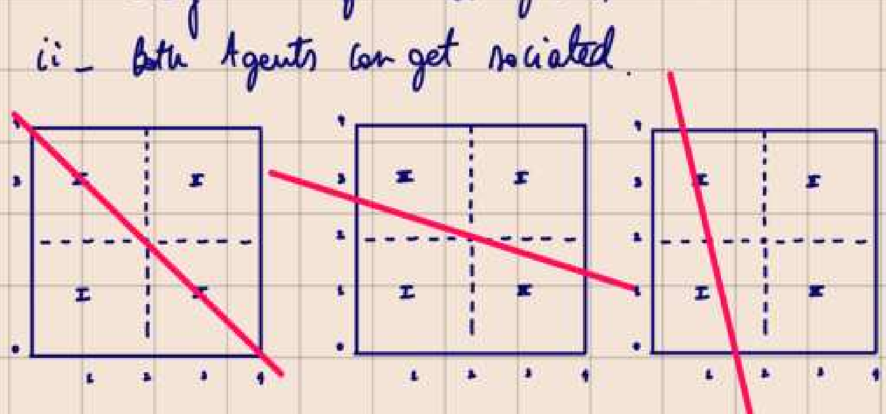
Then:

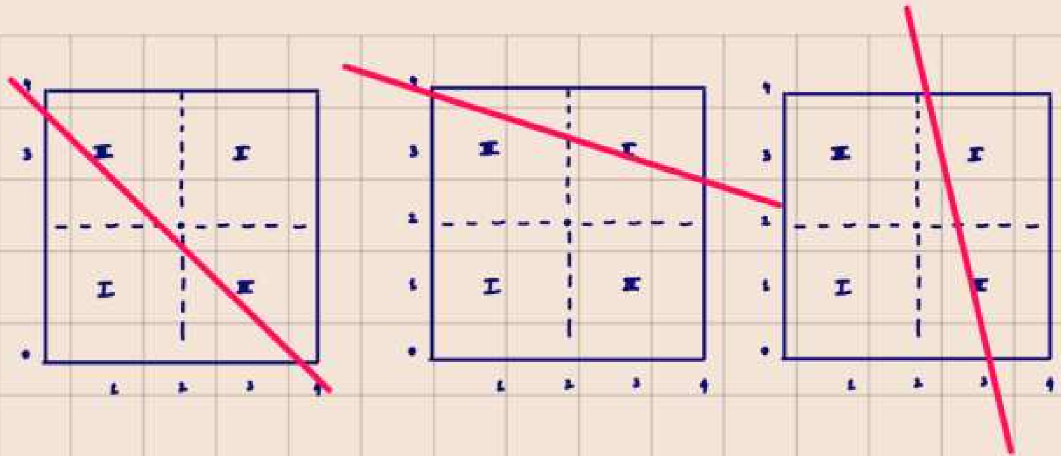
- ⓐ for both agents is a competitive equilibrium
- ⓐ for Agent 1 and ⓑ for agent 2 is a competitive equilibrium
- ⓐ for agent 1 and ⓒ for agent 2 is not a competitive equilibrium.
  (Excess demand of good 1)

To compute CE we can exploit the fact that one of 2 things happen:

i - Only 1 Agent can get satiated

ii - Both Agents can get satiated.





CASE 1

therefore, for any $(p_1, p_2) \in \mathbb{R}^2_{++}$ and any specification of endowments $(e_{11}, e_{12}, e_{21}, e_{22})$ st $\exists\ i \in \{1,2\}$ where

$$p_1 c_{i1} + p_2 c_{i2} < p_1 2 + p_2 2$$

and allocations given by optimal demands derived before

$$c_{21} = c_{21}^*(p_1, p_2, e_{21}, e_{22})$$
$$c_{22} = c_{22}^*(p_1, p_2, e_{21}, e_{22})$$
$$c_{11} = c_{11}^*(p_1, p_2, e_{11}, e_{12})$$
$$c_{12} = c_{12}(p_1, p_2, e_{11}, e_{12})$$

and $\quad c_{11} + c_{21} \leq 4$

$$c_{12} + c_{22} \leq 4$$

constitutes a competitive equilibria.

CASE 2   for any $(p_1, p_2) \in \mathbb{R}^2_{++}$ and any specification of endowments $(e_{11}, e_{12}, e_{21}, e_{22})$ st

$$p_1 c_{i1} + p_2 c_{i2} = p_1 2 + p_2 2 \quad \forall i:$$

and allocations given by optimal demands derived before

$$c_{21} = c_{21}^*(p_1, p_2, e_{21}, e_{22}) = 2$$
$$c_{22} = c_{22}^*(p_1, p_2, e_{21}, e_{22}) = 2$$
$$c_{11} = c_{11}^*(p_1, p_2, e_{11}, e_{12}) = 2$$
$$c_{12} = c_{12}(p_1, p_2, e_{12}, e_{12}) = 2$$

constitutes a competitive equilibra.

CASE 3   strange cases. Any endowment and:

- $(p_1, p_2) = (0, 0)$ and $(2,2,2,2)$ is a CE.

CASE 4 $(p_1, p_2) = (x, 0)$ where $x > 0$ and

$$c_{12} = 2, \qquad c_{i1} = \begin{cases} x, \ x \geq 2 & \text{if } e_{i1} > 2 \\ e_{i1} & \text{if } e_{i1} \leq 2 \end{cases} \quad \forall i$$
$$c_{22} = 2,$$

Is a CE

CASE 5   Similar case for $(p_1, p_2) = (0, y)$ $y > 0$

Final remark: $\exists$ CE that are not PE. Why does the FWT doesn't hold?