RECITATION 2: MICRO B102

Definition: A price/allocation (P, C) is a competitive equilibrium for a given specification of ownership $\{e_1, \dots, e_I\}$ if:

i) $p \in \mathbb{R}_+^M$

ii) $c_i \in C_i \quad \forall i$

iii) $\sum_i c_{i,m} \leq e_m \quad \forall m$ } feasible

iv) $\forall i, \ \sum_m p_m c_{im} \leq \sum_m p_m e_{mi}$ } affordable

v) $\forall i$ and $\hat{c}_i$ st $\hat{c}_i \succ_i c_i$, $\sum_m p_m \hat{c}_{im} > \sum_m p_m e_{im}$ } Individually optimal

strictly prefered bundles are not affordable.

Let $u_i(c_{i1}, c_{i2}) = \min\{2, c_{i1}\} + \min\{2, c_{i2}\}$ $\quad i=1,2$ and $e_1 = e_2 = 4$.

1. Characterize the UPS.
2. Characterize the set of PE allocation.
3. Are all solutions to PP PE?
4. Are all PE allocation solution to some PP?
5. Are all solution to Negishi(SPP) PE?
6. Are all PE allocation solution to some Negishi(SPP)?

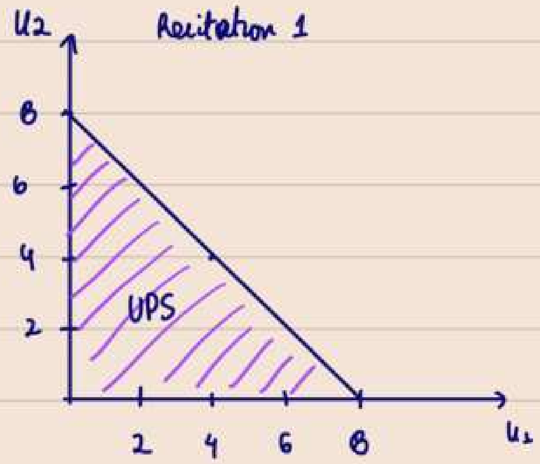
1. Characterize the set of CE allocation?
2. Are all CE PE?
3. Are all PE allocation CE for some endowment distribution? (SWT)

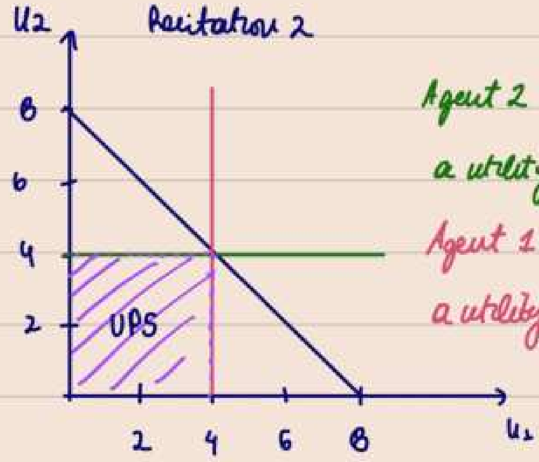
characterize the UPS. Graphical approach.

Note that if $c_{i1} \leq 2$ $c_{i2} \leq 2$ this agent would have a utility given by:

$$u_i(c_{i1}, c_{i2}) = c_{i1} + c_{i2}$$

By recitation 1, since $e_1 = e_4 = 4$

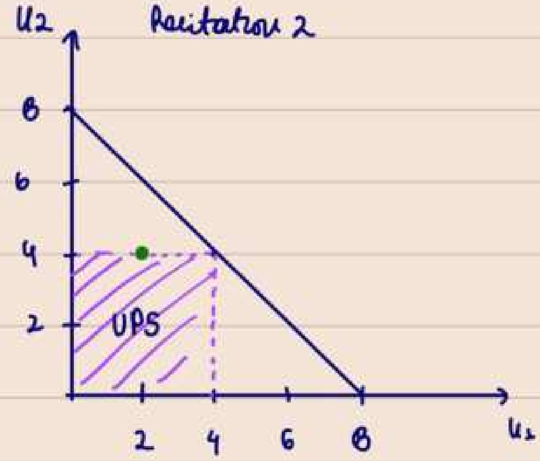




Agent 2 cannot have a utility greater than 4

Agent 1 cannot have a utility greater than 4

From UPS to allocations:



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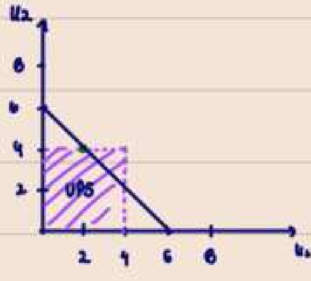
- $u_2=4$ and $u_1=2$. We know then that $c_{21}, c_{22} \geq 2$ and $c_{11}+c_{12}=2$.

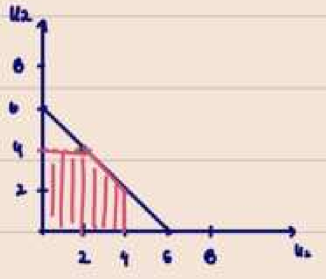
There are multiple allocations that generate this point

$$(c_{11}, c_{12}, c_{21}, c_{22}) = \begin{cases} (1, 1, 3, 3) \\ (2, 0, 2, 2) \\ (0, 2, 2, 2) \end{cases}$$

Note that markets don't even need to clear.

How would this change if for example $e_1=e_2=3$?





Doing this algebraically:

Note that if $\bar{u}_1 \in [0,4]$ then we can solve the PP for agent 2 given a specification for $\bar{u}_1$:

$$u_2(\bar{u}_1) = \max_{C} : \min\{c_{21}, 2\} + \min\{c_{22}, 2\}$$

$$\text{St:} \begin{cases} \min\{c_{11}, 2\} + \min\{c_{12}; 2\} \geq \bar{u}_1 \\ c_{11} + c_{21} \leq 4 \\ c_{12} + c_{22} \leq 4 \\ c_{11}, c_{12}, c_{21}, c_{22} \geq 0 \end{cases}$$

<u>Split into cases:</u>

If $u_1 = 4$, $c_{11} = c_{12} = c_{21} = c_{22} = 2$ with $u_2 = 4$.

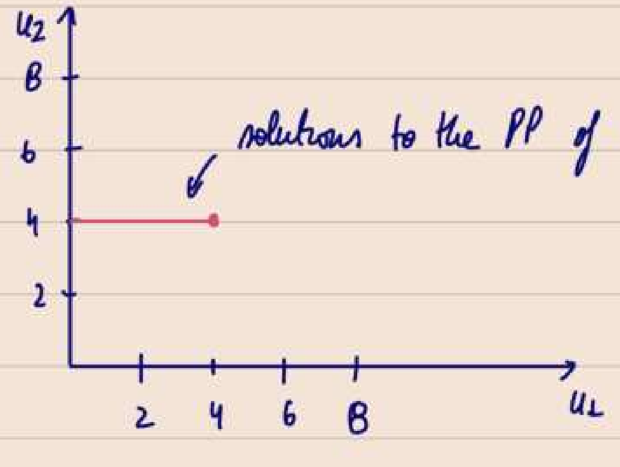
If $0 \leq u_1 < 4$; we have infinite solutions to the PP of agent 2. Why?

all the Pareto problem solutions for agent 2 given $\bar{u}_1 \in [0,4)$ are given by:

$$\left\{ (c_{11}, c_{12}, c_{21}, c_{22}) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : \begin{array}{l} c_{21} \geq 2 \quad c_{22} \geq 2 \\ c_{11} + c_{12} = \bar{u}_1 \end{array} \right\}$$

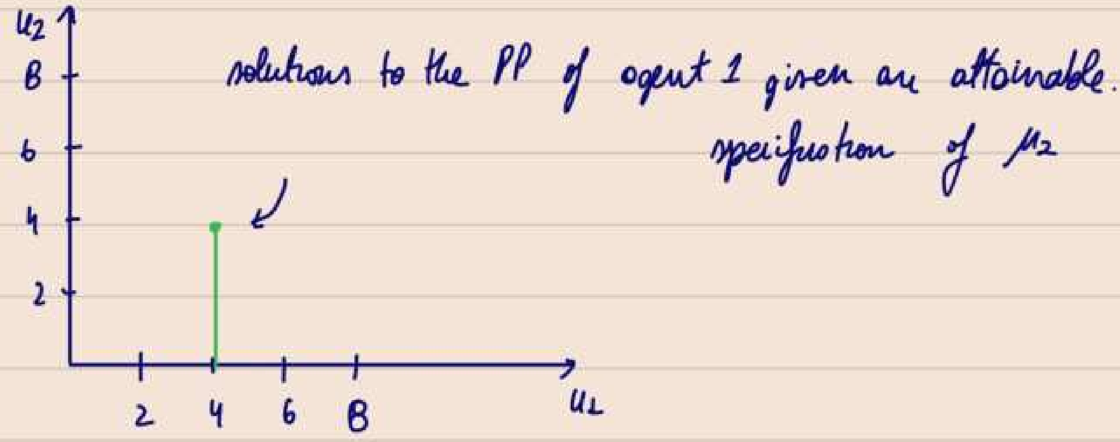
Even in this case $u_2(\bar{u}_1) = 4$.

Therefore, we know that $u_2(\bar{u}_1) = 4 \quad \forall \ \bar{u}_1 \in [0,4]$

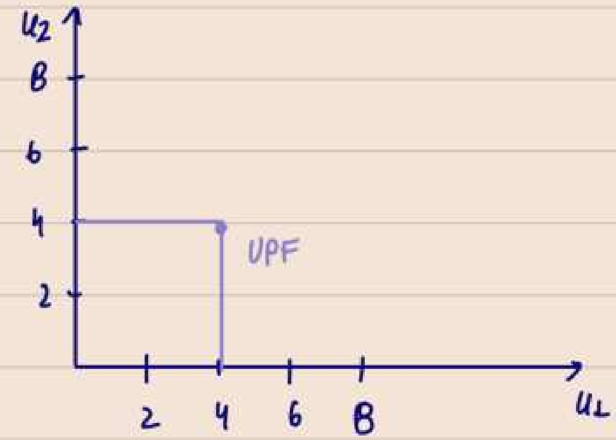


solutions to the PP of agent 2 given an attainable specification of u_1

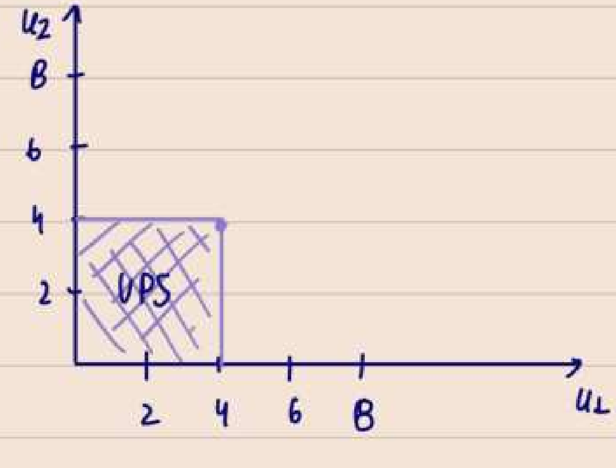
Repeating the analysis for agent 1;



Combining both results:



Going from the UPF to the UPS is easy in this exercise.



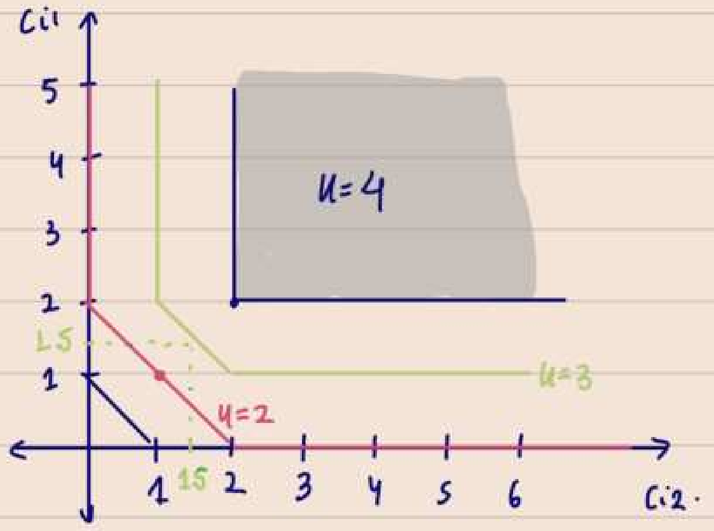
Are all points "inside" or "under" the UPF in the UPS? not always
For example when we work with floor preferences.

Characterize the set of PE allocations.

We know that if $C \in \{PE\}$ then $C \in \{PP\}$. Is checking every $\{PP\}$ to see if it is (PE) a good Idea? No, to many solutions.

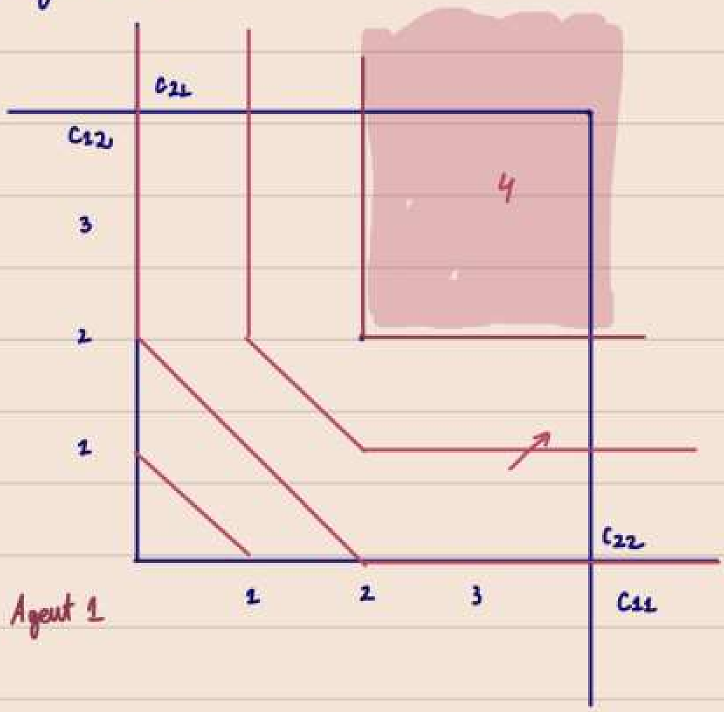
Can we state a theorem to show that $\{PP\} = \{PE\}$. No.
(which assumption of the proof doesn't hold?)

Graph the preferences for agent i.

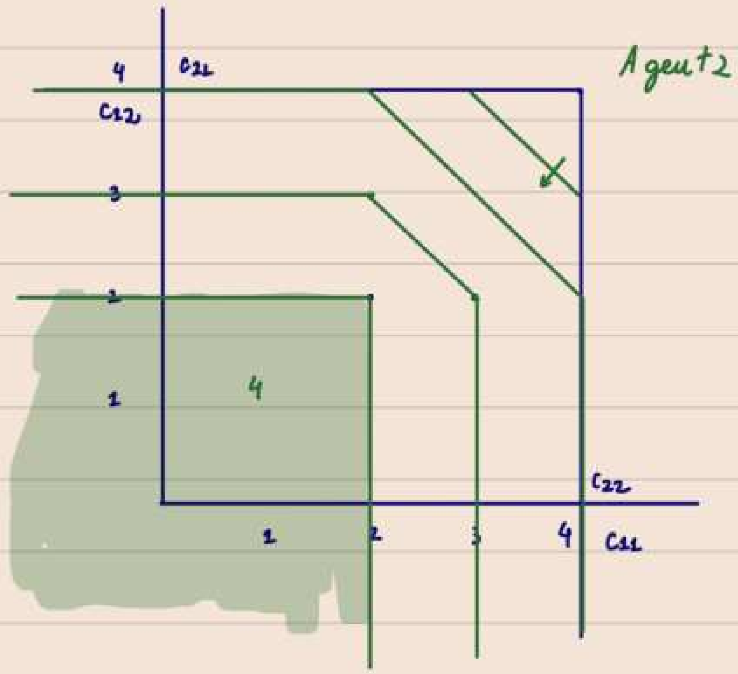


Then the edgeworth box can be drawn as:

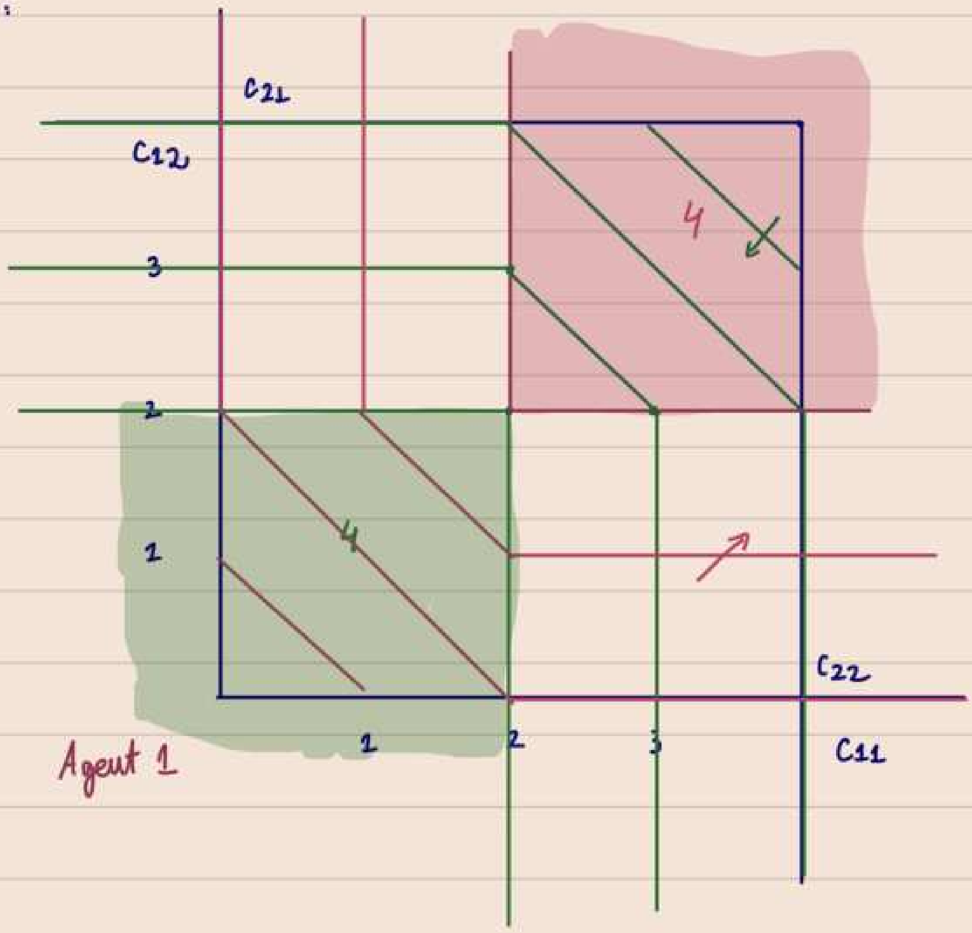
Drawing for agent 1:



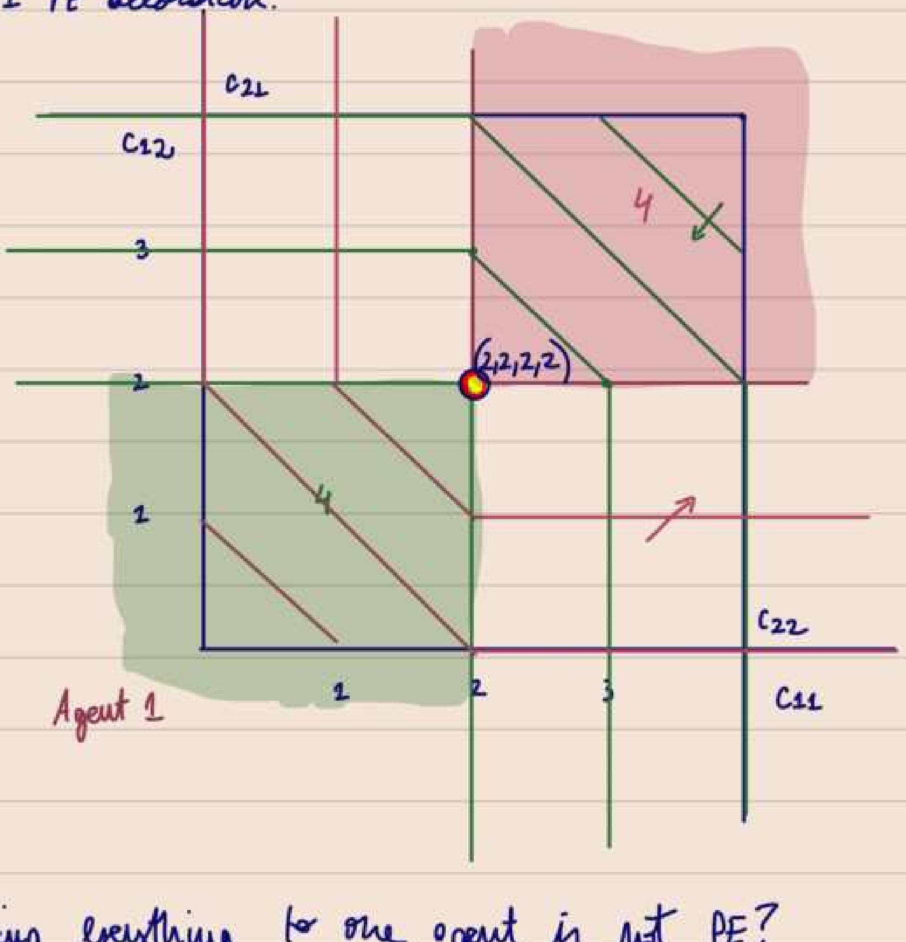
Drawing for Agent 2



Combining:

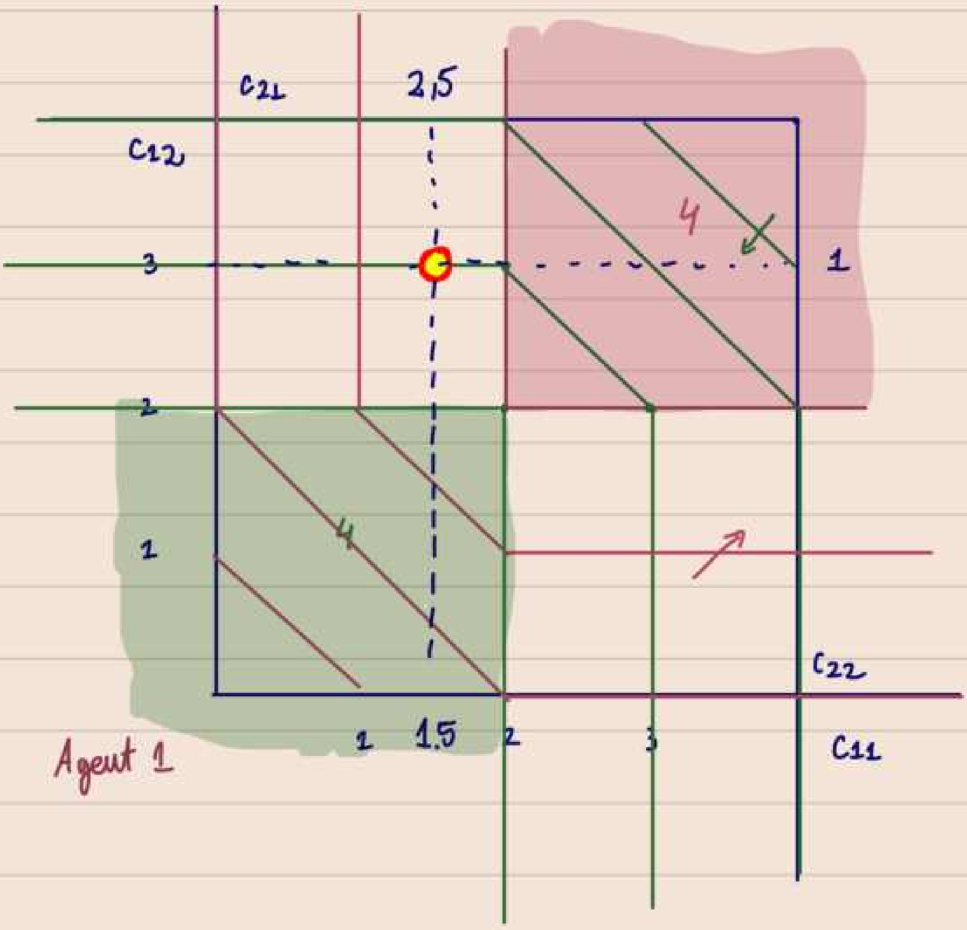


only 1 PE allocation:

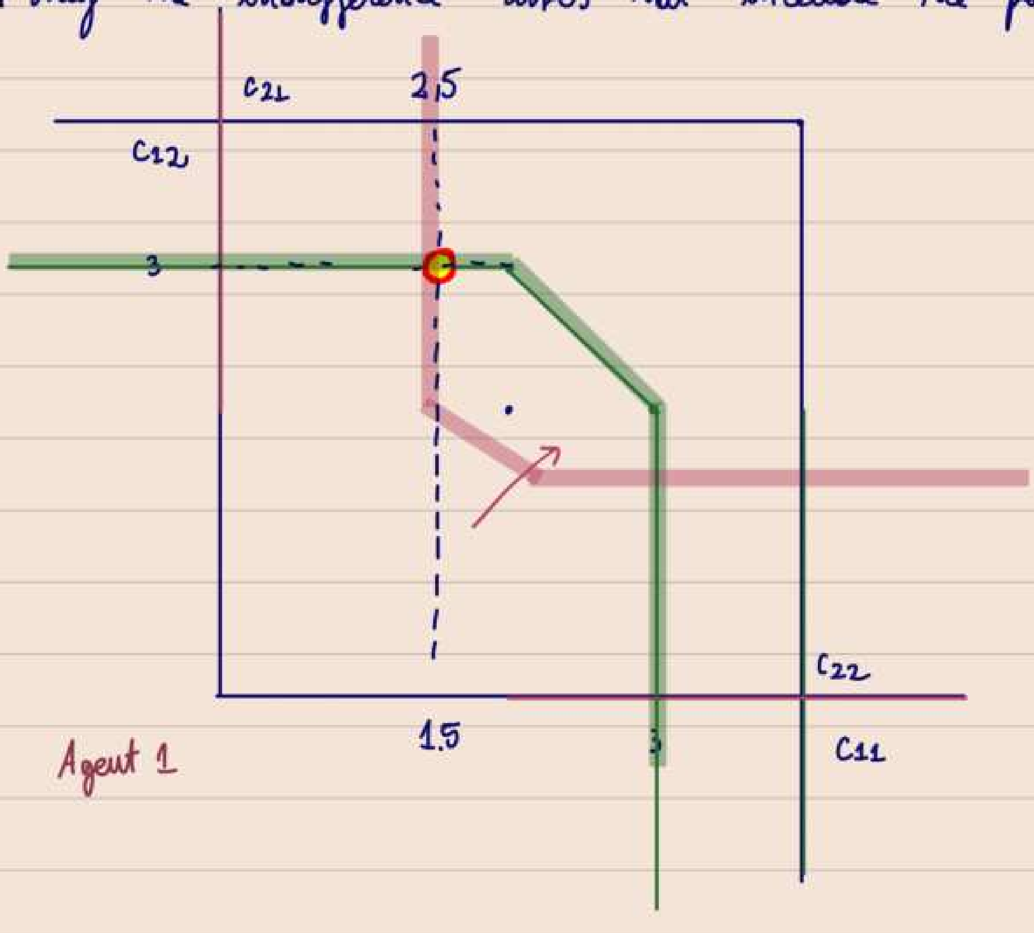


Why giving everything to one agent is not PE?

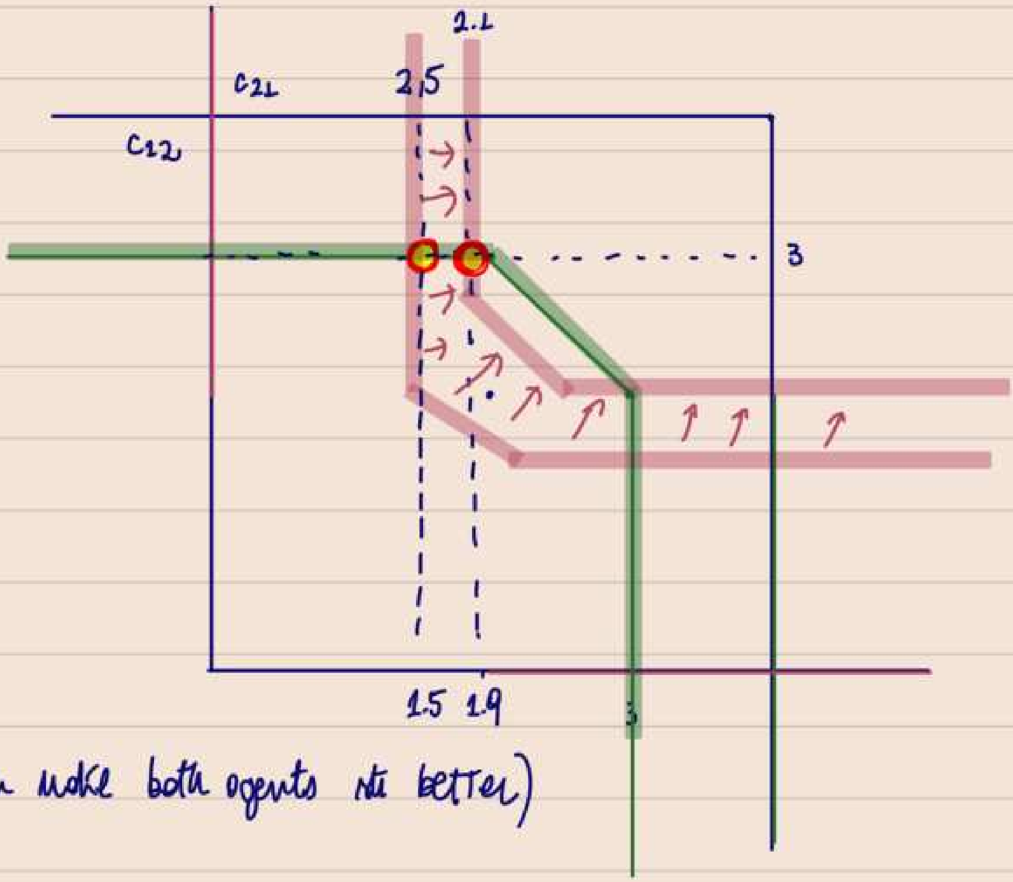
Take any other point, for example $C = (1.5, 3, 2.5, 1)$



consider only the indifference curves that include the point.



We can make agent 1 str. better without making 2 worst off.



(we could even make both agents str. better)

We know Agent 2 is not worst after the change since he is in the same indifference curve.

With this reasoning we can show that only (2,2,2,2) is PE.

Another way would be to think about the intersection of the solutions to the Pareto problem for both agents.

Solution to the SPP:

Given $\alpha \in [0,1]$, c is a solution to the SPP if it solves:

$$V(\alpha) = \max_{c} \ \alpha\left(\min\{c_{11}, 2\} + \min\{c_{12}, 2\}\right) + (1-\alpha)\left(\min\{c_{21}, 2\} + \min\{c_{22}, 2\}\right)$$

st

$$c_{11} + c_{12} \leq 4$$

$$c_{12} + c_{22} \leq 4$$

$$c_{11}, c_{22}, c_{21}, c_{22} \geq 0.$$

This is equivalent, (see Recitation notes 1):

$$V(\alpha) = \max_{(u_1, u_2)} \alpha u_1 + (1-\alpha) u_2$$
$$\text{st } (u_1, u_2) \in UPS$$

Then, since we care about the allocations that solve this problem, not only the value $V(\alpha)$, we go back from utilities to allocations. (We need to be careful not to forget that there might be multiple allocations that take you to the same point in the UPS.

If $\alpha = 0$; $SPP(\alpha=0) = \left\{ (c_{11}, c_{12}, c_{21}, c_{22}) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : \begin{array}{r} c_{11} \geq 2 \\ c_{12} \geq 2 \\ c_{11} + c_{21} \leq 4 \\ c_{21} + c_{22} \leq 4 \end{array} \right\}$

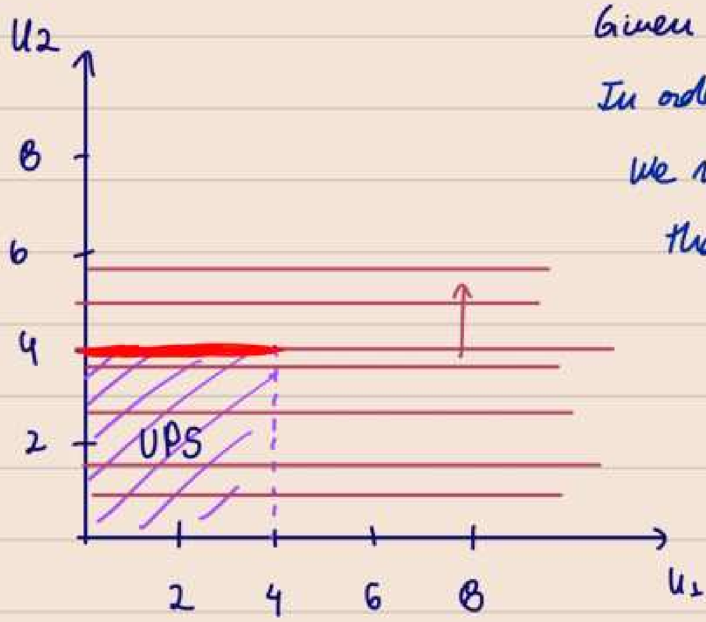
If $\alpha = 1$; $SPP(\alpha=1) = \left\{ (c_{11}, c_{12}, c_{21}, c_{22}) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : \begin{array}{r} c_{21} \geq 2 \\ c_{22} \geq 2 \\ c_{11} + c_{21} \leq 4 \\ c_{21} + c_{22} \leq 4 \end{array} \right\}$

If $\alpha \in (0,1)$: $SPP(\alpha \in (0,1)) = \left\{ (c_{11}, c_{12}, c_{21}, c_{22}) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : \begin{array}{r} c_{11} = 2 \\ c_{21} = 2 \\ c_{12} = 2 \\ c_{22} = 2 \end{array} \right\}$

We clearly see that there are solutions to the SPP that are not Pareto Efficient.

graphically:

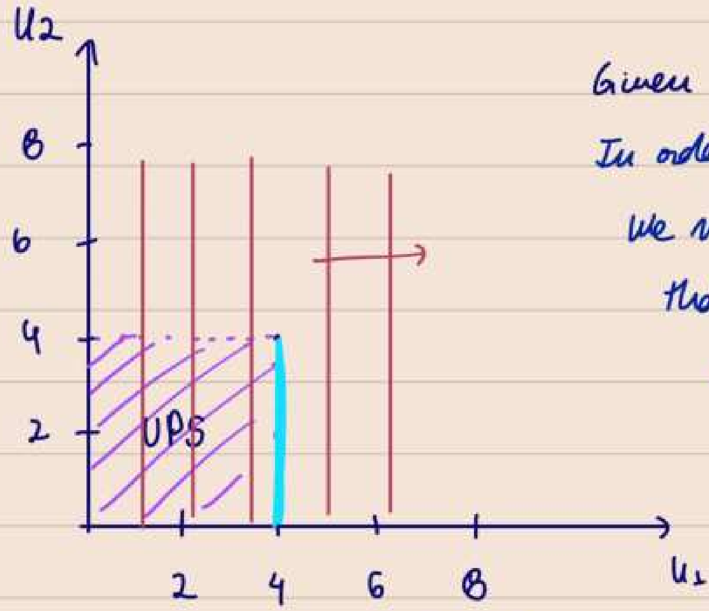
$\alpha = 0$



Given $\alpha = 0$:

In order to solve get the SPP solution we need to get the allocations that generate all this red combinations of (u_1, u_2)

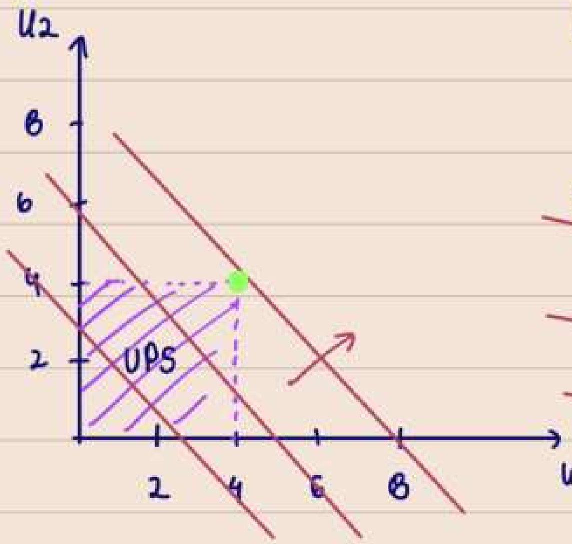
$\alpha = 1$

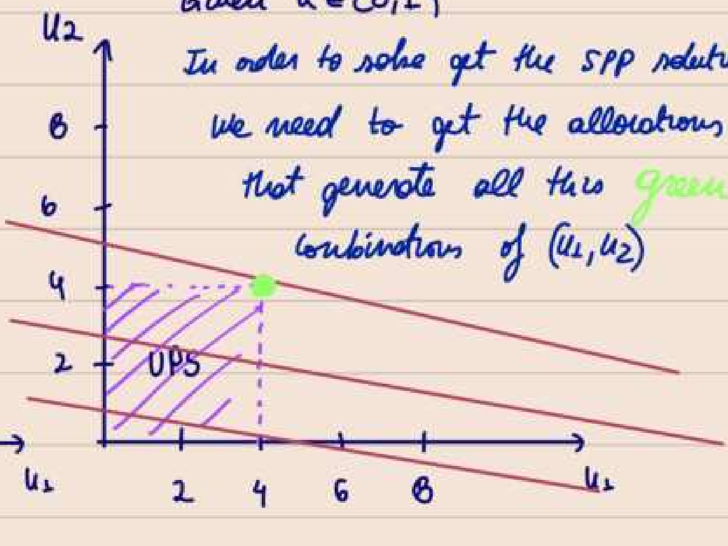


Given $\alpha = 1$:

In order to solve get the SPP solution we need to get the allocations that generate all this cyan combinations of (u_1, u_2)

For any $\alpha \in (0,1)$, For example for two different α's:





Given $\alpha \in (0,1)$

In order to solve get the SPP solutions we need to get the allocations that generate all this green combinations of (u_1, u_2)