$\boxed{\text{To complete last recitation}}$

<u>Recap:</u> The set of Pareto problem solutions:

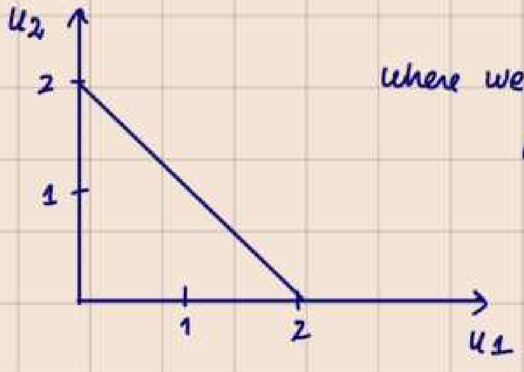
$$PP = PE = \left\{ (c_1, c_2) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ : \begin{array}{l} c_{11} + c_{21} = 1 \\ c_{12} + c_{22} = 1 \end{array} \right\}$$

Now, lets compute all the solutions to the Negishi problem (SPP)

Take any $\alpha \in [0,1]$; c is a solution to the Negishi problem if it solves:

$$\max_{\{c\}} : \alpha\, u_1(c_1) + (1-\alpha)\, u_2(c_2)$$
$$c_{11} + c_{21} \leq 1$$
$$c_{12} + c_{22} \leq 1$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

$\boxed{\text{Method 1}}$ Draw the UPS.



where we can write the UPF as:

$$u_2(u_1) = 1 - u_1 \quad \text{for} \quad u_1 \in [0,2]$$

Note that solving the (SPP) is equivalent to:

$$\max \ \alpha\, u_1 + (1-\alpha)\, u_2$$
$$(u_1, u_2) \in UPS$$

(And then once we obtain the (u_1, u_2) that max the expression we go back to allocations for us to give the final answer)

<u>If $\alpha = 1$</u> we are just solving:

(I would always recommend analysing the case where $\alpha=1$ and $\alpha=0$ separately)

$$\max \ u_1$$
$$\text{st } (u_1, u_2) \in UPS$$

<u>If $\alpha=0$</u> we are just solving

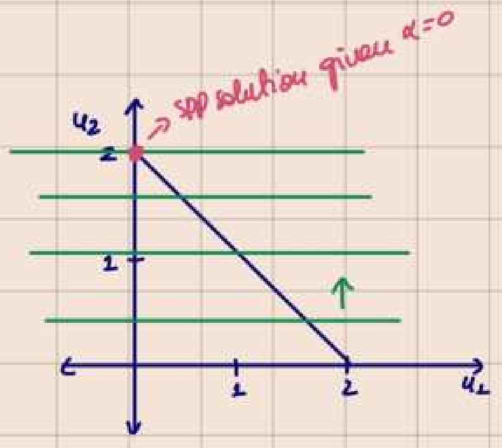
$$\max \ u_2$$
$$\text{st } (u_1, u_2) \in UPS$$

If $\alpha \in (0,1)$, Note that for any given value of the objective function $\bar{D}$, we can rewrite this as:

(we could also show that if $\alpha \in (0,1)$ the allocation that solves the SPP is PE)

$$\bar{D} = \alpha \ u_1 + (1-\alpha) \ u_2$$
$$-(1-\alpha) \ u_2 = \alpha \ u_1 - \bar{D}$$
$$u_2 = \bar{D} - \frac{\alpha}{1-\alpha} u_1$$

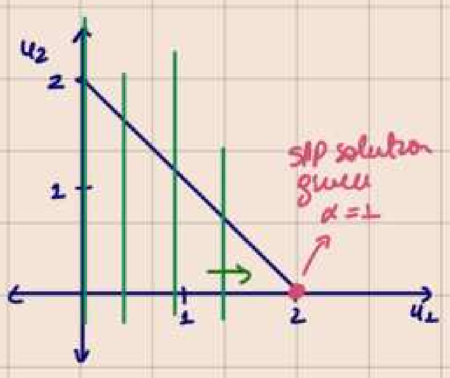
<u>Graphically:</u>



If $\alpha=0$ we are solving

$\max \ u_2$

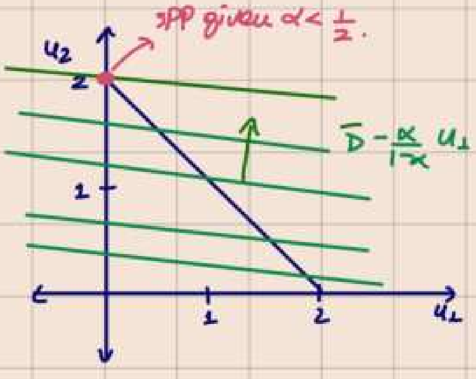
st $(u_1,u_2) \in UPS$



If $\alpha=1$ we are solving

$\max \ u_1$

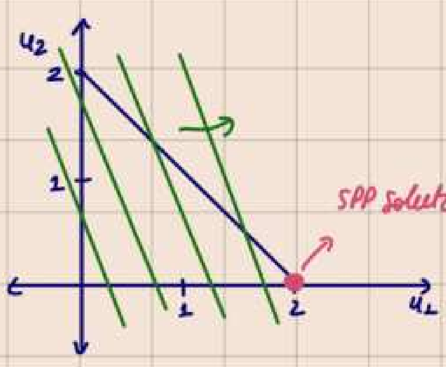
$(u_1, u_2) \in UPS$



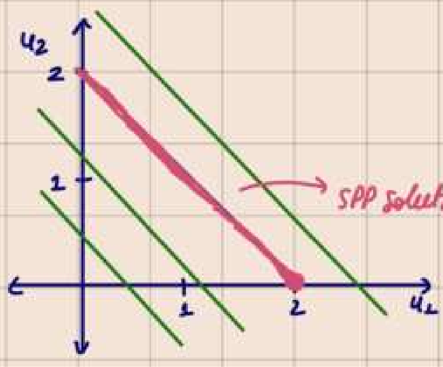
If $\alpha \in (0,1)$ we are solving

$\max \alpha u_1 + (1-\alpha) u_2$

$(u_1, u_2) \in UPS$.



SPP solution if $\alpha > \frac{1}{2}$.



SPP solution if $\alpha = \frac{1}{2}$

If $\alpha \in (0,1)$ we are solving

$\max \alpha u_1 + (1-\alpha) u_2$

$(u_1, u_2) \in UPS$.

If $\alpha \in (0,1)$ we are solving

$\max \alpha u_1 + (1-\alpha) u_2$

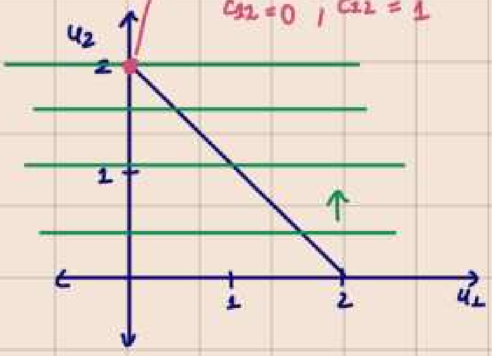
$(u_1, u_2) \in UPS$.

(note that in this case the UPS has slope $\left(-\frac{\alpha}{1-\alpha}\right)$)

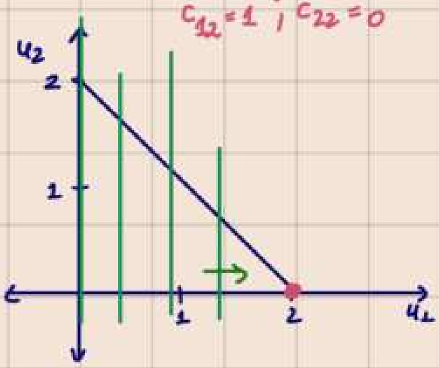
we found what is the value of the objective but not the allocations that solve for those values. we do that now.

This corresponds to allocations

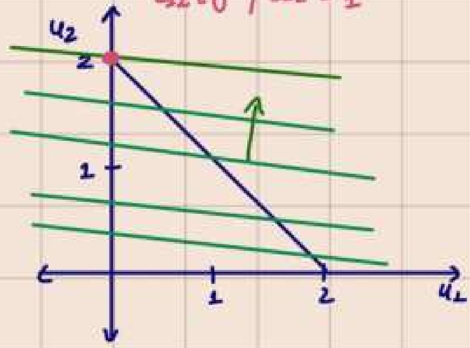
$c_{11} = 0$; $c_{21} = 1$
$c_{12} = 0$; $c_{22} = 1$

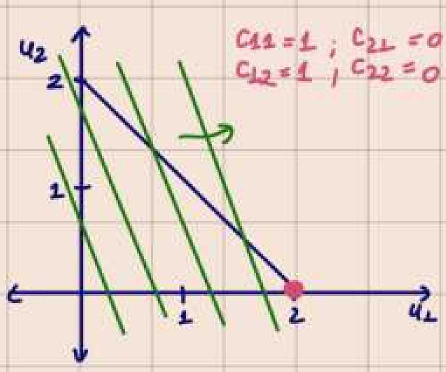


$c_{11} = 1$; $c_{21} = 0$
$c_{12} = 1$; $c_{22} = 0$

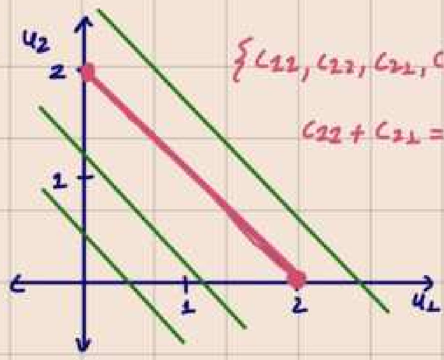


$c_{11} = 0$; $c_{21} = 1$
$c_{12} = 0$; $c_{22} = 1$





$c_{11} = 1$; $c_{21} = 0$
$c_{12} = 1$; $c_{22} = 0$



$\{c_{11}, c_{21}, c_{21}, c_{22} \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 :$

$c_{11} + c_{21} = 1$; $c_{12} + c_{22} = 1\}$.

⇒ we see that in this easy example $\{SPP\} = \{PE\} = \{PP\}$.

Another way of solving for this without drawing:

for any $\alpha \in [0,1]$; c is a solution to the Negishi problem if it solves

$$\max_{\{c\}} \quad \alpha\, u_1(c_1) + (1-\alpha)\, u_2(c_2)$$
$$c_{11} + c_{21} \leq 1$$
$$c_{12} + c_{22} \leq 1$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

$$\max_{\{c\}} \quad \alpha\,(c_{11} + c_{12}) + (1-\alpha)\,(c_{21} + c_{22})$$
$$c_{11} + c_{21} \leq 1$$
$$c_{12} + c_{22} \leq 1$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

Since both u_1 and u_2 are str increasing in consumption c

$$\max_{\{c\}} \quad \alpha\,(c_{11} + c_{12}) + (1-\alpha)\,(c_{21} + c_{22})$$
$$c_{11} + c_{21} = 1$$
$$c_{12} + c_{22} = 1$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

If this was not here what would happen for $\alpha < \frac{1}{2}$?

If $\alpha = \frac{1}{2}$: $\quad \max_{\{c\}} \quad \frac{1}{2}\,(c_{11} + c_{21}) + \frac{1}{2}\,(c_{12} + c_{22})$
$$c_{11} + c_{21} = 1$$
$$c_{12} + c_{22} = 1$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

Any allocation that satisfies the constraints will maximize the objective.

$$= \underset{\{c\}}{\operatorname{argmax}}: \alpha (c_{11} + c_{22}) + (1-\alpha)(1 - c_{11} + 1 - c_{12})$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

$$= \underset{\{c\}}{\operatorname{argmax}}: (1-\alpha) \cdot 2 + \alpha (c_{11} + c_{12}) - (1-\alpha)(c_{11} + c_{22})$$
$$c_{11}, c_{12}, c_{21}, c_{22} \geq 0$$

$$= \underset{\{c\}}{\operatorname{argmax}}: (1-\alpha) \cdot 2 + \alpha (c_{11} + c_{12}) - (1-\alpha)(c_{11} + c_{12})$$
$$1 \geq c_{11} \geq 0 \quad 1 \geq c_{12} \geq 0$$

$$= \underset{\{c\}}{\operatorname{argmax}} \ (2\alpha - 1)(c_{11} + c_{22})$$
$$1 \geq c_{11} \geq 0 \quad 1 \geq c_{12} \geq 0$$

(The constant won't change the argmax.)

Note that if $\alpha > \frac{1}{2}$ (social planner cares more about agent 1) $(2\alpha - 1) > 0$ and the expression is maximized when $c_{11} = c_{12} = 1$. Since $c_{11} + c_{21} = 1$ and $c_{12} + c_{22} = 1$, we know $c_{21} = c_{22} = 0$.

If $\alpha < \frac{1}{2}$ (social planner cares more about agent 2) and the expression is maximized when $c_{11} = c_{12} = 0$. Since $c_{11} + c_{21} = 1$ and $c_{12} + c_{22} = 1$ with $c_{21} \geq 0$ and $c_{22} \geq 0$; we know $c_{21} = c_{22} = 0$.

Exercise: Repeat the analysis for the case
(Final 2021)

$$\begin{cases} u_1(c_{11}, c_{12}) = \min\{c_{11}, 2\} + \min\{c_{12}, 2\} \\ u_2(c_{21}, c_{22}) = \min\{c_{21}, 2\} + \min\{c_{22}, 2\} \\ e_1 = e_2 = 4 \end{cases}$$

+ Find the CE for any specification of individual endowments

⇒ Recitation 2