Last Recitation

Fall 2018

Ask → Speed
↳ Detail

Assumptions:

$\textcircled{s}$ $\xrightarrow{\pi}$ H: Agent 1, 2 bananas; Agent 2, 1 bananas

$\textcircled{s}$ $\xrightarrow{1-\pi}$ T: Agent 1, 0 Bananas; Agent 2, 1 Bananas.

- no production, all endowments are observable.
- $s \in \{H, T\}$.
- $\begin{cases} e_1(H) = 2 & e_1(T) = 0 \\ e_2(H) = 1 & e_2(T) = 1. \end{cases}$
- Preferences of Agent 1: $\pi \sqrt{c_1(H)} + (1-\pi)\sqrt{c_1(L)}$
- Preferences of Agent 2: $\pi\, c_2(H) + (1-\pi)\, c_2(L)$
- $G_1 = \mathbb{R}_+^2 \quad G_2 = \mathbb{R}^2$

{ Before you start this exercise 2 things that are easy to miss that are important (At least for me).

1° - one Agent is risk neutral.

2° - $G_2 = \mathbb{R}$.

3° - there is Agg risk }

ⓐ {An allocation is how much each person consumes for each possible realization of history}

$$c = \left(c_1(H), c_1(T), c_2(H), c_2(T) \right) \in \mathbb{R}_+^2 \times \mathbb{R}^2.$$

An feasible allocation is an allocation st:

(sumations)

$$c_1(H) + c_2(H) \leq e_1(H) + e_2(H)$$

$$\sum_i c_i(H) \leq \sum_i e_i(H)$$

and

$$c_1(T) + c_2(T) \leq e_1(T) + e_2(T)$$

$$\sum_i c_i(T) \leq \sum_i e_i(T)$$

or:

$$\sum_i c_i(s) \leq \sum_i e_i(s) \quad \forall \quad s \in \{H, T\}$$

Constructing the PE allocations. {This is the same exercise that we did before midterm if we think of $c_1(H)$ and $c_2(T)$ as being different goods at the same point in time}

[Since both agents have continuous and str. monotone utility functions we know that {PE = PP}.]

We solve the PP for Agent 1

- As always I want to start offering feasible values for $\bar{u}_2$
- Note that if agent 2 consumes everything in the economy for each realization,

$$c_2(H) = 3$$
$$c_2(T) = 1$$

So since Agent 1 cannot consume negative the maximum value u_2 can get is $\pi\, c_2(H) + (1-\pi)\, c_2(T) =$

$$\pi \quad 3 + (1-\pi) \, 1 = \bar{u}_{2MAX}$$
$$\pi\, 3 + 1 - \pi = (1 + 2\pi) = \bar{u}_{2MAX}$$

But Is the minimum 0? No, It is $-\infty$...

$\rightarrow$ we can make $c_2(H), c_2(T) \longrightarrow -\infty$ (It's feasible) (It's able to consume this)

Here $u_2 \in (-\infty, \overline{u_2}^{MAX}]$. Now let's set-up the PP for 1: ④

$$u_1(\overline{u_2}) = \max_{\{c_1(H), c_1(T), c_2(H), c_2(T)\}} \quad \pi \sqrt{c_1(H)} + (1-\pi)\sqrt{c_1(T)}$$

$$\text{st:} \quad \pi\, c_2(H) + (1-\pi)\, c_2(T) \geqslant \overline{u_2}$$

$$c_1(H) + c_2(H) \leqslant 3$$

$$c_1(T) + c_2(T) \leqslant 1$$

$$c_1(H),\ c_2(T) \geqslant 0 .\ !.$$

First let's simplify this problem:

$$c_1(H) + c_2(H) = 3$$

$$c_1(T) + c_2(T) = 1$$

$$\pi\, c_2(H) + (1-\pi)\, c_2(T) = \overline{u_2}$$

therefore.

$$c_1(H) = 3 - c_2(H)$$

$$c_1(T) = 1 - c_2(T)$$

and we can rewrite the problem as:

$$u_1(\overline{u_2}) = \max_{c_2(H),\ c_2(T)} \quad \pi\sqrt{3 - c_2(H)} + (1-\pi)\sqrt{1 - c_2(T)}$$

$$\pi\, c_2(H) + (1-\pi)\, c_2(T) = \overline{u_2}$$

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(5)

FOC

Let λ be the Lagrange multiplier associated with the only constraint of this problem.

$$\pi \ \frac{1}{2} \ \frac{1}{\sqrt{3-c_2(H)}} = \pi \lambda \qquad (1)$$

$$(1-\pi) \ \frac{1}{2} \ \frac{1}{\sqrt{1-c_2(T)}} = (1-\pi)\lambda . \qquad (2)$$

Combining (1) and (2):

$$\frac{1}{\sqrt{3-c_2(H)}} = \frac{1}{\sqrt{1-c_2(T)}}$$

$$3-c_2(H) = 1-c_2(T)$$

$$-c_2(H) = -2-c_2(T)$$

$$c_2(H) = 2 + c_2(T) .$$

In the constraint:

$$\pi \ \overbrace{\left(2 + c_2(T)\right)}^{c_2(H)} + (1-\pi) \ c_2(T) = \overline{u_2}$$

$$\pi \ 2 + c_2(T) = \overline{u_2}$$

$$c_2(T) = \overline{u_2} - 2\pi \quad (\text{Alom!})$$

$$c_H(T) = (2-2\pi) + \overline{u_2}$$

we can check that this makes sense

then from feasibility

$$c_1(H) = 3 - c_2(H)$$

$$c_1(H) = 3 - (2 - 2\pi) - u_2$$

$$c_1(H) = (1 + 2\pi) - u_2$$

In the other state:

$$c_1(T) = 1 - c_2(T)$$

$$c_1(T) = 1 - (\bar{u}_2 - 2\pi)$$

$$c_1(T) = (1 + 2\pi) - \bar{u}_2$$

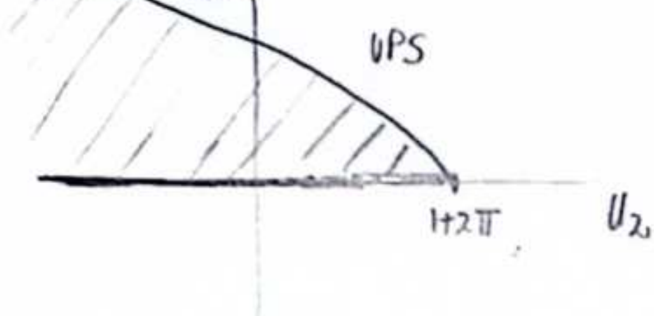
check when the agent consumes zero of both goods? It holds

then we can draw the UPF and UPS as:

$$u_1(\bar{u}_2) = \pi\sqrt{(1+2\pi) - u_2} + (1-\pi)\sqrt{(1+2\pi) - \bar{u}_2}$$

for $u_2 (-\infty ; \bar{u}_{max})$

where $u_2 \in (-\infty, \bar{u}_{2MAX})$



Agent 2 Is risk neutral and does not care about 7
consuming negative. {In any Pareto efficient allocation the risk averse agent consumes constant across states at the expense of the risk neutral agent}

All pareto efficient allocations are given by:

$$\left.\begin{aligned} c_1(H) &= (1+2\pi) - u_2 \\ c_1(T) &= (1+2\pi) - u_2 \\ c_2(H) &= u_2 - 2\pi \\ c_2(T) &= (2-2\pi) + u_2 \end{aligned}\right\}$$

$PE = \{ c_1(H), c_1(T), c_2(H), c_2(T) \in \mathbb{R}_+^2 \times \mathbb{R}^2 :$
$c_1(H) = c_1(T) = c_1$ and
$c_1(H) + c_2(H) = e_1(H) + e_2(H)$
$c_1(T) + c_2(T) = e_1(H) + e_2(T) \}$

for any $u_2 \in (-\infty; \overline{u}_{max}]$.

This is one way of solving the problem, another one would be solving the SPP for Agent 2. Another one would involve arguing (somehow) that the UPF is strictly convex then we know that $\{SPP = PE\}$
We will do that now..

SPP, given weights $\mu \in [0,1]$

$$\max_{c_1(H), c_1(T), c_2(H), c_2(T)} \mu\left(\pi\sqrt{c_1(H)} + (1-\pi)\sqrt{c_1(T)}\right) + (1-\mu)\left(\pi\, c_2(H) + (1-\pi)\, c_2(T)\right)$$

$$\text{st:} \quad c_1(H) + c_2(H) \leq 3$$

$$c_1(T) + c_2(T) \leq 1$$

$$c_1(H),\ c_1(T) \geq 0$$

- If $\mu = 1$ then there is no solution.
- If $\mu = 0$ then $c_1(H) = c_1(T) = 0$ agent 2 consumes everything in the economy. $c_2(H) = 3$, $c_2(T) = 1$
- If $\mu \in (0,1)$; we know that feasibility will hold with equality but I think it's easier to solve the problem with $\lambda(H)$ and $\lambda(T)$ lagrange multipliers associated with the constraints.

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(We should include constraints associated with $c_1(H), c_1(T) \geq 0$ but I know they won't hold.)

$$[c_1(H)]: \quad \mu\pi\frac{1}{2}\frac{1}{\sqrt{c_1(H)}} = \lambda(H)$$

$$[c_2(H)]: \quad (1-\mu)\pi = \lambda(H)$$

$$[c_1(T)]: \quad \frac{1}{2}\mu(1-\pi)\frac{1}{\sqrt{c_1(T)}} = \lambda(T)$$

$$[c_2(T)]: \quad (1-\mu)(1-\pi) = \lambda(T)$$

mbining all of these conditions:

$$\mu \cancel{\pi} \frac{1}{2} \frac{1}{c_1(H)} = (1-\mu) \cancel{\pi}$$

$$\frac{1}{2} \frac{1}{c_1(H)} = \frac{(1-\mu)}{\mu}$$

$$c_1(H) = \frac{1}{2} \frac{\mu}{1-\mu}.$$ (check $\uparrow\mu$ $c_1(H)\uparrow$ ✓)

Doing the same for $S=T$;

$$\mu \cancel{(1-\pi)} \frac{1}{2} \frac{1}{c_1(T)} = (1-\mu) \cancel{(1-\pi)}$$

$$c_1(T) = \frac{1}{2} \frac{\mu}{1-\mu}$$

by feasibility, any PE allocation would be given by:

$$c_1(H) = c_1(T) = c_1$$

and $c_2(H) = 3 - c_1$

$c_2(T) = 1 - c_1$

for any value of c_1 !

The set of PE allocations is given by

$$c^{PE} = \{ (c_1(H), c_1(T), c_2(H), c_2(T)) \in \mathbb{R}_+^2 \times \mathbb{R}^2 : \quad c_1(H) = c_1(T) \}$$

$$c_2(H) = 3 - c_1(H)$$
$$c_2(T) = 1 - c_1(T)$$

(c) characterize the CE:

In this type of Models the risk neutral Agent will pin down prices. (Is like the CRS firm) because since we are allowing him to consume negative, he can sell consumption of one good to buy consumption of the other good

Assumption: Arrow-Debreu market structure. People meet before the coin is flipped and they trade contingent claims, taking the prices as given. They don't know if Tails or heads will land but they know what they will do if T or H.

Agent 2's problem: (Before Anything is realized goes to the market and sees prices $P(H), P(T)$.)

$$\max_{c_2(H), c_2(T)} : \pi \, c_2(H) + (1-\pi) \, c_2(T)$$

$$\text{st} \quad P(H) \, c_2(H) + P(T) \, c_1(T) = P_1(H) \, e_2(H) + P_2(T) \, e_2(H)$$

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Xin's question!

$$
\left.\begin{array}{ll}
[C_2(H)]: & \pi = P(H)\,\lambda \\
[C_2(T)]: & (1-\pi) = P(T)\cdot\lambda
\end{array}\right\} \quad \frac{\pi}{1-\pi} = \frac{P(H)}{P(T)} =
$$

{Else this agent would choose to consume $-\infty$ in some state to consume $+\infty$ in another state and this will break feasibility} we cannot pin this agents consumption yet, only prices (CRS fun)! We have Prices In equilibrium!

think about why this is true!

Solve Agent 1's problem, taking as given prices:

$$
\max_{C_1(H),\,C_1(T)} \quad \pi\sqrt{C_1(H)} + (1-\pi)\sqrt{C_1(T)}
$$

$$
P(H)\,C_1(H) + P(T)\,C_1(T) = P(H)\,\underbrace{e_1(H)}_{2} + P(T)\,\underbrace{e_1(T)}_{0}
$$

$$
C_1(H) \geq 0,\; C_1(T) \geq 0.
$$

FOC

$$
\left.\begin{array}{ll}
[C_1(H)]: & \pi\,\frac{1}{2}\frac{1}{\sqrt{C_1(H)}} = \lambda\,P(H) \\
[C_1(T)]: & (1-\pi)\,\frac{1}{2}\frac{1}{\sqrt{C_1(T)}} = \lambda\,P(T)
\end{array}\right\} \quad \frac{\pi}{1-\pi}\frac{\sqrt{C_1(T)}}{\sqrt{C_1(H)}} = \frac{P(H)}{P(T)}
$$

(USING CE PRICES) $\boxed{C_1(T) = C_1(H) = C_1}$

Therefore FIRST Welfare the

therefore $c_1(H) = c_1(T) = c_1$. The Agent want to smooth consumption given those prices. (If prices were not this one we might not have this result)"

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$$P(H)\ c_1 + P(T)\ c_1 = P(H)\ 2 + P(T)\ 0$$

$$\frac{P(H)}{P(T)}\ c_1 + \frac{P(T)}{P(T)}\ c_1 = \frac{P(H)}{P(T)}\ 2 + \frac{P(T)}{P(T)}\ 0$$

$$\left(\frac{\pi}{1-\pi} + 1\right) c_1 = \frac{\pi}{1-\pi}\ 2.$$

$$\frac{1}{1-\pi}\ c_1 = \frac{\pi}{1-\pi}\ 2.$$

$$\boxed{c_1 = \pi \cdot 2}$$

π is the probability we land in the state where 1 has all his wealth. As this state becomes more likely to happen, the Agent is richer and consumes more.

CE is given by relative prices $\frac{P(H)}{P(T)} = \frac{\pi}{1-\pi}$ and consumption allocations $\left(c_1(H), c_1(T), c_2(H), c_2(T)\right) = \left(2\pi, 2\pi, 3-2\pi, 1-2\pi\right)$