

8101 - Recitation IV: Household Demand and Theorem of Afriat

Exercise 1

Consider stationary recursive utility on infinite-time bounded consumption plans with no uncertainty, that is $(c_0, c_1, \dots) \in \ell_+^\infty$, defined by the aggregator

$$G(z_1, z_2) = z_1 + \beta \ln(z_2 + 1)$$

for $z_1 \geq 0$ and $z_2 \geq 0$, and the linear period-utility $u(x) = x$.

1. Show that there is unique time-independent utility $U(d)$ for every constant consumption plan $d_t = d$, for every $d \geq 0$.

Exercise 2

Consider the utility function on $\mathbb{R}_+^3$ given by $u(c_1, c_2, c_3) = c_1 + \sqrt{c_2 c_3}$. Show that u does not have separable representation.

Exercise 3

Consider preference relation $\succsim$ defined on the set of non-negative contingent claims R_+^S on a state-space with S states of nature. Assume that $\succsim$ is continuous and strictly increasing.

1. State necessary and sufficient condition(s) for $\succsim$ to have state-separable utility representation when $S \geq 3$.
2. Prove that the condition you stated in (1) is indeed necessary.
3. Does the condition you stated in (1) remain necessary and sufficient in the case of two states, i.e., $S = 2$? Justify your answer.