8101 - Recitation IV: Household Demand and Theorem of Afriat

Exercise 1

Consider a household with two members A and B. There are two goods: Good 1 is for private consumption of each member while good two is for household's "public" consumption. Utility functions are

$$u_A(x_A, z) = 2\ln(x_A) + z, \quad \text{and } u_B(x_B, z) = \ln(x_B) + 2z,$$

where $x_A > 0$, and $x_B > 0$ denote private consumption of good 1 by the respective member, and $z \geq 0$ denotes the household's public consumption of good 2. Joint consumption plan of two goods is $y = (x_A + x_B, z)$.

1. Derive the household utility function of joint consumption y.
2. Derive the unitary-household demand function f that maps prices p_1 and p_2, joint income w, and a Pareto weight $0 < \mu < 1$ to joint consumption.
3. Derive the Slutsky matrix of f for arbitrary (p, w) and $\mu = \frac{1}{2}$. Verify that the Slutsky matrix is negative semi-definite and symmetric.
4. Consider a non-unitary household with weight μ given by $\mu(p, w) = \frac{p_2}{w}$, and restrict your attention to (p, w) such that $p_2 < w$. Derive the non-unitary household demand function.

Exercise 2

Consider the utility function u on $\Re^L_+$ in the proof of the Theorem of Afriat, 13.1, in Lecture Notes. That is,

$$u(x) = \min_{t=1,\dots,T} \left\{u^t + \lambda^t p^t \left(x - x^t\right)\right\},$$

where $\{(p^t, x^t)\}_{t=1}^T$ is a finite collection of pairs of price vectors and consumption plans such that $p^t \in \Re^L_{++}$ and $x^t \in \Re^L_+$, and $\{(u^t, \lambda^t)\}_{t=1}^T$ are positive numbers. Show that u is continuous, concave, and increasing.