8101 - Recitation III: Revealed preferences and Monotone comparative statics

Exercise 1

1. Show that if a rational preference relation $\succeq$ is continuous and locally non-satiated then $p.\ x^*(p,w)=w$, for all $x^*(p,w)$ that belong to the Walrasian Demand correspondence.

2. Consider Walrasian demand function $x^*(p,w)$ assumed single valued - for $p \gg 0$ and $w \geq 0$ generated by a continuous and locally nonsatiated utility function $u: R_+^L \rightarrow R$. State and prove the Law of Compensated Demand for x^*.

Exercise 2:

Consider a list of observations$\{(p_1,x_1),\ldots,(p_T,x_T)\}$ where $p_t \in \mathbb{R}_+^N$ and $x_t \in \mathbb{R}_+^N$ are price vector and a corresponding consumption plan of a consumer respectively, for every $t \in \{1,\ldots,T\}$.

1. State the Generalized Weak Axiom of Revealed Preference (GWARP) and Generalized (strong) Axiom of Revealed Preference (GARP) for these observations.

2. Show that if a locally non-satiated utility function rationalized observations then GWARP holds.

3. Suppose that the observations are generated by a demand function $d(p,w)$ that is $x_t = d(p_t,w_t)$ for every t. Function d is given as

$$d(p,w)=\begin{cases}\left(\frac{w}{p_1},0\right) & \text{if } p_1 \geq p_2\\ \left(\frac{w}{p_1+p_2},\frac{w}{p_1+p_2}\right) & \text{if } p_2>p_1\end{cases}$$

   Does GWARP hold for arbitrary observations generated by d ? Can demand d be rationalized by a locally non-satiated utility function?

Exercise 3:

Consider the problem of finding a Pareto optimal allocation of aggregate resources $\omega \in \Re_+^n$ in an economy with two agents:

$$\max_x \mu_1 u_1(x)+\mu_2 u_2(\omega-x)$$

subject to $x \leq \omega, x \geq 0$ where $u_i: \Re_+^n \rightarrow \Re$ are agents' utility functions (assumed continuous) and $\mu_i>0$ are welfare weights for $i=1,2$. Let $x^*(\mu_1,\mu_2)$ be the set of solutions.

1. Show that if u_i is supermodular, then $u_i(\omega-x)$ is supermodular in x.

2. Show that, if u_1 and u_2 are strictly increasing and supermodular in x , then $x^*(\mu_1,\mu_2)$ is non-decreasing in μ_1. You may assume that $x^*(\mu)$ is single-valued. Is$x^*(\mu_1,\mu_2)$ non-increasing in μ_2 ? Justify your answer.

3. Under what conditions on u_1 and u_2 is the solution $x^*(\mu)$ unique. Justify.